MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXV. APRIL 14, 1905. No. 6

W. H. MAW, Esq., PRESIDENT, in the Chair.

William Bowyer, Royal Observatory, Greenwich; and 12 Maidenstone Hill, Greenwich, S.E.;
Rev. Thomas Joseph Charlton, The Rectory, Omeath, co. Louth, Ireland;
Capt. Louis Arthur Demers, Marine and Fisheries Department, Ottawa, Canada;
David James Reginald Edney, Royal Observatory, Greenwich, and Teston Lodge, Blackheath Rise, Lewisham, S.E.;
Herbert Henry Furner, Royal Observatory, Greenwich; and 7 Circus Street, Greenwich, S.E.;
John Adelbert Parkhurst, M.Sc., Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.;
Montagu Austin Phillips, F.R.G.S., F.Z.S., F.E.S., 22 Petherton Road, Highbury New Park, N.; and
Arthur L. Wood, H.M.S. "Conway," Rock Ferry, Birkenhead,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Major B. F. S. Baden-Powell, 32 Prince's Gate, S.W. (proposed by W. J. S. Lockyer);
Joseph Henry Elgie, 72 Grange Avenue, Leeds (proposed by C. T. Whitmell);

Frederick William Longbottom, Haslemere, Queen's Park, Chester (proposed by W. E. Plummer) ;
Frederick John Marrian Stratton, B.A., Isaac Newton Student in the University of Cambridge, Caius College, Cambridge (proposed by Sir R. S. Ball) ; and
John Willis, late India Office, retired, Merkara, 19 Bouverie Square, Folkestone (proposed by Sir R. S. Ball).

The following were proposed by the Council as Associates of the Society :—

Theodor Albrecht, Königlich-preussische geodätische Institut, Berlin-Potsdam, Germany ;
Gustav Müller, Astrophysikalisches Observatorium, Potsdam, Germany ; and
Rodolphe Radau, Membre de l'Institut, Paris.

Seventy-one presents were announced as having been received since the last meeting, including, amongst others :—

Astronomical Discovery, by H. H. Turner, presented by the author ; 19 charts of the Astrographic Chart of the Heavens, presented by the Royal Observatory, Greenwich, and 18 charts presented by the San Fernando Observatory ; Sydney Observatory, Observations, 1862, presented by the Observatory.

Magnetic Disturbances as recorded at the Royal Observatory, Greenwich, and their Association with Sun-spots. Second Paper. By E. Walter Maunder.

4. *Magnetic Disturbances*, 1848 *to* 1881.

It was my wish after the completion of my former paper on the magnetic disturbances of 1882–1903 (*Monthly Notices*, vol. lxv. No. 1, pp. 2–34) to extend the same inquiry to all the years for which there are regular magnetic records at Greenwich. In the ordinary way this inquiry would have entailed a great amount of work and occupied a considerable time. But my work has been greatly facilitated by Mr. William Ellis, F.R.S., who generously placed at my disposal his lists of disturbed days for fifty-five years, 1848–1902. I was thus saved the need of consulting the records in their entirety ; it was only necessary for me to refer to the photographic sheets for the dates which he had indicated, and the days immediately preceding and following them. Nor was this my only gain ; for I had the advantage of Mr. Ellis's unequalled experience in the estimation

of the importance of the disturbances, whilst the fact that the selection had been wholly his, and had been made long before my paper, and therefore wholly independently of it, prevented the possibility of any bias in favour of what I may call the "Interval-Relation" having had anything to do with their selection.

In the following catalogue, therefore, the second and third columns are copied precisely from Mr. Ellis's catalogue, and the grouping of different disturbed days into continuous storms is also entirely his. In no case have I permitted myself to make any alteration in his list, although, since it was made for quite a different purpose from that for which I am now using it, in some cases an alteration of his grouping might legitimately have been made. I have only reproduced here the first thirty-four years of the entire collection of fifty-five which Mr. Ellis prepared, as the last twenty-one years have already been dealt with in Table I. of my former paper, and the differences between his catalogue for these years and mine are quite unimportant. In a few cases where he has included disturbances of a slightly smaller amplitude or simpler character than those exhibited in the Greenwich Plates the sequences brought out in Table I. are extended and completed, and so the evidence for the relation, to which I have called attention, is somewhat strengthened; but in general the two catalogues are in very close accord for the years which they have in common.

Dr. Charles Chree, F.R.S., has very kindly given me every facility for consulting the Kew registers for the occasions upon which the Greenwich registration was at fault. The catalogue is therefore complete, with the exception of five days before the Kew photographic records were commenced.

In the second column disturbances in which the greatest amplitude of movement in declination exceeded one degree of arc are indicated by the letter "G" for "great"; those movements exceeding thirty minutes are indicated by the letter "A" for "active"; the remaining disturbances are classed by Mr. Ellis as "moderate," with maximum movements of about twenty minutes up to thirty. As Mr. Ellis's catalogue only gave the days of disturbance, and not the times at which the disturbances began and ended, I had to take these out from the original sheets to the best of my judgment, and for the fourth and fifth columns of the catalogue I am therefore responsible. I have also added, as in Table I. of my earlier paper, the number of the solar rotation, and the heliographic longitude of the centre of the Sun's disc; the same numeration of the rotations, the same prime meridian and rotation period, being adopted as in the Heliographic Results given in the annual volumes of the Greenwich observations. For the earlier years I have numbered the rotations backwards from the Greenwich fundamental rotation—namely, that commencing 1853 November 9, which is No. 1 in the Greenwich numeration. The Sun's sidereal period of

rotation has been assumed to be 25·38 days, corresponding to a mean synodic rotation of 27·275 days; the longitude of the ascending node has been assumed as 73° 40′ for 1850·0, and the meridian which passed through the ascending node at the epoch 1854·0 has been taken as the prime meridian. The times in the catalogue are Greenwich mean solar time throughout, since Mr. Ellis took the astronomical day, not the civil day, as his unit. Hence the times should be increased by twelve hours to make them comparable with those given in Table I. of my earlier paper, wherein I used Greenwich civil time.

TABLE IX.

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1848.					
1		Jan. 14	14	6	7	−77	344·2
2	A	16	16	2	13	...	320·0
3	A	28	28	4	19	...	161·0
4		Feb. 14	14	8	7	−76	294·9
5	A	20	20	0	...	...	220·3
...	G	21	...		...	...	...
...		22	...		...	...	...
...		23	...		...	...	...
...	A	24	...		122	...	...
6	A	Mar. 19	No reg.		...	−75	205 ±
...	A	20	...		{No. reg.}	...	...
7		Apr. 2	1	12	31	...	33 3
8		6	No. reg.		...	−74	327 ±
...	A	7	...		{No. reg.}	...	...
9		May 9	9	18	...	−73	248·1
...		10	...		16	...	...
10		17	17	4	...	...	150·0
...	A	18	...		39	...	...
11	A	July 11	10	13	29	−71	150·4
12		Sept. 8	8	9	7	−69	79·3
13	G	Oct. 18	17	14	24	−67	281·9
14	A	23	23	3	17	...	208·8
15		25	25	0	17	...	184·1
16	A	27	27	4	7	...	155·5
17	A	29	29	12	...	...	124·8
...		30	...		24	...	...

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1848.					
18	G	Nov. 17	16	22	...	−66	241·9
...	A	18	...		45	...	...
19	A	21	21	5	9	...	185 4
20		26	26	10	14	...	116·7
21	A	Dec. 17	17	5	16	−65	202 8
		1849.					
22		Jan. 14	13	20	26	−64	198·9
23		16	16	6	3	...	167·1
24		Feb. 18	18	6	...	−63	92·6
...		19	...		...	...	...
...		20	...		58	...	...
25	A	22	22	5	10	...	40·5
26		26	26	10	...	−62	345·1
...	A	27	...		34	...	...
27		Mar. 30	30	14	2	−61	281·1
28		Sept. 24	24	6	6	−55	92·2
29	A	Oct. 14	14	5	6	−54	188·9
30	A	22	21	20	15	...	88·3
31	A	30	30	8	11	−53	336·3
32	A	Nov. 27	27	8	...	−52	327·1
...		28	...		...	...	...
...	A	29	...		64	...	...
		1850.					
33	A	Feb. 23	22	7	28	−49	261·8
34		Mar. 31	31	7	10	−48	134·2
35		Apr. 6	6	11	5	...	52·8
36		June 13	13	7	9	−45	236·0
37		July 11	11	10	...	−44	223·7

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1850.					
...		July 12	...		30	...	...
38		Oct. 1	1	1	...	−41	225·1
...		2	...		41	...	...
		1851.					
39	A	Jan. 16	16	1	16	−37	254·7
40	A	19	19	9	20	...	210·8
41		Feb. 5	5	8	6	−36	347·6
42	A	18	18	2	16	...	179·7
43		July 14	14	7	13	−31	48·1
44	A	Aug. 24	24	6	4	−29	226·5
45	A	Sept. 3	3	18	...	...	87·8
...	A	4	...		27	...	...
46	A	6	6	2	...	...	56·9
...	G	7	...		41	...	...
47		27	27	1	11	−28	140·3
48	G	29	29	3	20	...	112·8
49	A	Oct. 1	1	7	...	...	84·2
...	G	2	...		38	...	...
50	A	28	28	9	14	−27	86·9
51		Nov. 11	11	9	4	−26	262·3
52	G	Dec. 6	6	7	10	−25	293·9
53		28	28	3	...	...	6·3
...	A	29	...		42	...	...
		1852.					
54		Jan. 4	3	22	48	−24	276·8
55	A	19	19	11	...	...	72·2
...		20	...		36	...	...
56	G	Feb. 14	14	21	...	−23	84·3
...	A	15	...		27	...	...
57	G	17	17	0	...	...	47·0
...	A	18	...		...	...	...
...	G	19	...		71	...	...
58	A	21	20	23	16	...	4·2
59	A	Mar. 26	26	5	20	−21	272·9
60		31	31	7	11	...	205·8
61		Apr. 8	8	7	4	...	100·3
62	A	20	20	3	...	−20	304·0
...		21	...		...	...	...

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1852.					
...	A	Apr. 22	...		66	...	...
63	A	May 27	27	5	18	−19	173·3
64	A	June 11	11	4	13	−18	335·7
65		July 10	10	3	13	−17	312·4
66		Sept. 29	{No reg.	No reg.}		−14	317 ±
67	A	Oct. 18	18	4	3	...	70·8
68		20	20	0	17	...	46·6
69	G	Nov. 11	11	7	18	−13	112·7
70		13	13	2	21	...	89·1
71		Dec. 10	10	6	12	−12	91·0
72		23	23	8	10	−11	278·7
73		29	{No reg.	No reg.}	...		197 ±
		1853.					
74		Jan. 10	10	4	9	...	43·8
75		Mar. 7	6	17	...	− 9	32·4
...		8	...		62	...	...
76		17	17	7	6	− 8	252 9
77		Apr. 5	5	2	48	...	5·1
78	A	May 24	24	5	18	− 6	75·9
79	A	June 22	22	0	17	− 5	54·9
80	G	July 12	12	1	20	− 4	149·6
81	A	Sept. 1	1	14	...	− 2	188·1
...	A	2	...		...	...	...
...		3	...		54	...	...
82		Oct. 31	31	4	7	0	121·8
83	A	Nov. 9	9	3	20	...	3·7
84	A	Dec. 6	6	0	24	+ 1	9·5
85	A	21	20	22	7	+ 2	172·9
		1854.					
86		Jan. 2	{No reg.	No reg.}	...		7 ±
87		20	19	11	22	3	143·9
88		29	29	4	12	...	16·1
89		Feb. 15	15	4	...	4	152·2
...	A	16	...		57	...	...
90	G	24	24	5	...	...	33·1
...	A	25	...		56	...	...

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1854.					
91	G	Mar. 15	15	7	...	5	141·7
...	A	16	...		33	...	...
92		26	26	7	7	6	356·7
93	A	28	27	21	20	...	335·8
94	A	Apr. 10	10	11	22	...	156·6
95		19	19	5	...	...	41·0
...		20	...		34	...	...
96		23	22	7	...	...	0·3
...		24	...		55	...	...
97	A	July 10	10	3	15	9	37·4
98		Sept. 11	10	14	22	12	291·7
99		26	26	9	14	...	83·2
100	A	Oct. 8	8	3	12	13	288·2
101		12	12	4	14	...	234·8
		1855.					
102		Feb. 8	8	4	12	17	106·8
103	A	Mar. 12	12	2	15	18	46·4
104		17	17	6	...	19	338·3
...		18	...		36	...	...
105		Apr. 4	4	4	10	...	102·0
106		12	12	10	9	20	353·1
107		June 22	22	6	18	22	136·5
108		July 19	19	5	18	23	139·6
109		Oct. 4	4	10	12	26	199·5
110		18	18	5	9	...	17·6
		1856.					
111		Apr. 22	22	11	6	33	69·8
112		Aug. 23	22	5	27	38	259·2
113		Sept. 8	8	9	8	...	32·5
114		Oct. 22	22	5	9	40	174·1
115		Dec. 3	3	4	5	42	340·9
		1857.					
116	G	May 7	7	2	20	47	99·0
117	A	Sept. 3	3	6	17	52	322·6
118		Nov. 12	12	6	9	54	119·1
119		16	16	4	...	...	67·4
...		17	...		53	...	...
120	G	Dec. 16	16	1	...	55	33·7
...		17	...		37	...	...

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1858.					
121	A	Jan. 8	7	23	25	56	91·9
122	A	Feb. 16	16	7	...	58	294·0
...		17	...		34	...	...
123		Mar. 5	5	6	3	...	70·6
124		12	12	0	...	59	341·7
...	A	13	...		...	...	...
...	A	14	...		...	...	...
...		15	...		104	...	...
125		17	17	5	12	...	273·0
126		28	28	17	31	...	121·4
127	G	Apr. 9	8	13	...	60	338·4
...		10	...		53	...	...
128		May 7	7	9	34	61	317·5
129	A	June 22	22	9	...	62	69·0
...	A	23	...		..	...	...
...		24	...		63	...	...
130		July 5	4	3	44	63	273·2
131		Aug. 25	25	1	21	65	306·7
132		Sept. 20	20	4	...	66	321·7
...		21	...		...	...	...
...		22	...		60	...	...
133		Oct. 17	17	8	6	67	323·3
134	A	27	27	12	12	...	189·2
135		31	31	6	30	...	139·7
136	A	Dec. 4	4	9	22	68	49·9
137		24	24	2	11	69	150·2
		1859.					
138	A	Feb. 9	9	5	...	71	249·7
...		10	...		28	...	...
139		22	22	11	...	...	75·2
...		23	...		46	...	...
140		26	26	1	11	71	28·0
141	A	Apr. 21	21	2	...	73	35·3
...	A	22	...		46	...	...
142		28	28	11	...	74	297·0
...	A	29	...		34	...	...
143		May 19	18	16	32	...	30·7

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1859.					
144	A	June 8	7	19	29	75	124·4
145	A	July 11	10	13	35	76	50·9
146	G	Aug. 28	27	20	...	78	132·3
...	A	29	...		52	...	...
147	G	Sept. 1	1	2	...	...	76·1
...	G	2	...		...	...	...
...	G	3	...		...	...	...
...	A	4	...		...	...	...
...		5	...		130	...	...
148		24	24	1	8	79	133·0
149		Oct. 1	1	7	...	...	37·3
...		2	...		34	...	...
150		4	4	4	10	80	359·4
151	G	12	12	14	24	...	248·3
152	A	17	17	8	...	...	185·7
...		18	...		49	...	...
153		20	20	9	...	...	145·6
...		21	...		34	...	...
154		Dec. 5	5	6	34	82	260·8
155	A	13	12	21	16	...	160·3
		1860.					
156	A	Feb. 21	21	0	35	85	316·8
157		Mar. 12	12	1	34	...	52·8
158		27	27	17	...	86	206·2
...	A	28	...		...	...	...
...	A	29	...		54	...	...
159	A	Apr. 9	8	21	24	...	45·7
160	A	13	12	8	40	...	0·1
161		June 10	10	13	9	89	297·2
162	A	29	29	11	...	...	46·8
...	A	30	...		...	...	...
...	A	July 1	...		61	...	...
163	A	4	4	3	...	90	345·0
...		5	...		42	...	...
164	A	11	11	10	14	...	248·5
165		19	19	11	10	...	142·1
166	A	Aug. 6	6	16	...	91	261·3

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1860.					
...	A	Aug. 7	...		...	...	...
...	A	8	...		...	...	...
...	A	9	...		...	...	...
...	G	10	...		...	...	...
...	A	11	...		...	...	...
...	G	12	...		167	...	...
167		16	16	2	19	...	136·8
168		30	30	2	19	92	311·8
169	G	Sept. 6	6	8	...	...	216·0
...	A	7	...		...	...	...
...		8	...		48	...	...
170		15	15	9	24	...	96·6
171		Dec. 10	10	0	24	95	47·5
172		16	15	12	30	96	335·0
		1861.					
173	A	Jan. 22	22	5	...	97	198·4
...		23	...		...	...	...
...	A	24	...		...	...	...
...	A	25	...		...	...	...
...	A	26	...		...	...	...
...		27	...		125	...	...
174	A	Feb. 27	26	23	...	98	87·6
...	A	28	...		49	...	...
175		Mar. 9	9	5	20	99	312·6
176		Apr. 15	15	3	19	100	185·7
177		Aug. 18	17	18	...	105	337·2
...		19	...		40	...	...
178	A	Oct. 10	10	21	19	107	342·6
179		24	23	20	30	...	171·7
180		Nov. 7	7	3	32	108	343·2
181	A	Dec. 19	19	3	...	109	149·7
...		20	...		37	...	...
		1862.					
182		Jan. 13	13	7	...	110	178·2
...		14	...		...	...	...
...	A	15	...		65	...	...
183	A	Feb. 21	20	21	17	111	30·2
184		Mar. 6	5	19	18	112	220·0

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1862.					
185		Apr. 2	2	2	12	113	220·2
186		10	10	8	30	...	111·4
187	G	July 5	5	5	8	116	55 6
188	A	7	7	10	7	...	26·4
189		23	23	6	15	117	176·8
190	A	Aug. 3	3	17	...	...	25·3
...	G	4	...		...	...	...
...	A	5	...		41	...	...
191	A	Sept. 24	23	23	14	119	68·2
192	G	Oct. 3	3	7	...	120	305·1
...		4	...		...	...	...
...		5	...		...	...	...
...	A	6	...		76	...	...
193	A	22	21	8	25	...	67·1
194	A	Nov. 18	17	9	...	121	70·5
...		19	...		39	...	..
195	A	Dec. 14	14	4	13	122	77·4
		1863.					
19		Jan. 8	8	4	12	123	108·1
197		10	10	10	4	...	78·5
198		12	12	3	6	...	56·0
199	A	24	24	7	...	124	255·8
...		25	...		...	...	...
...	A	26	...		52	...	...
200		29	29	4	13	...	191·6
201		31	31	2	12	...	166·4
202		Feb. 7	6	15	...	...	80·2
...	A	8	...		48	...	...
203		22	22	8	11	125	233·4
204	A	25	25	5	...	...	195·5
...		26	...		32	...	...
205		Mar. 21	21	6	10	126	238·7
206		Apr. 8	8	5	24	...	1·8
207		15	15	6	18	127	268·9
208		July 15	15	9	11	130	143·5
209		Aug. 13	13	10	...	131	119·4
...		14	...		38	...	...

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1863.					
210		Aug. 29	28	2	33	132	285·5
211	A	Sept. 9	9	6	...	...	124·8
...		10	...		40	...	...
212		23	23	6	...	133	300·0
...		24	...		30	...	...
213	A	Oct. 8	7	8	52	...	114·2
214		Nov. 6	5	7	28	134	92·3
215		14	14	1	...	135	336·9
..		15	...		36	...	...
		1864.					
216		Feb. 1	1	6	12	137	13·5
217		11	11	8	9	138	240·7
218		Mar. 6	6	6	32	139	285·7
219	A	10	10	3	...	...	234·6
...		11	...		40	...	...
220	A	Apr. 27	27	6	32	141	319·6
221		May 5	5	6	11	...	213·9
222		25	25	6	10	142	309·3
223	A	June 7	7	4	28	...	138·4
224	A	22	22	16	...	143	293·2
...		23	...		46	...	...
225	A	July 19	18	15	9	144	309·7
226		Aug. 14	13	7	31	145	330·2
227		Sept. 17	16	19	14	146	234·5
228	A	20	20	10	...	...	186·6
...		21	...		26	...	...
229		23	22	7	30	...	161·9
230	A	Oct. 13	13	5	...	147	245·9
...		14	...		34	...	...
231		19	19	3	9	...	167·9
232		Nov. 11	11	2	10	148	225·1
233		15	15	1	15	...	173·0
234		Dec. 12	12	2	7	149	176·6
235		15	15	1	10	...	137·6
236		23	23	3	10	...	31·1
		1865.					
237		Jan. 11	11	5	3	150	139·8
238		16	16	4	...	...	74·5

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1865.					
...		Jan. 17	...		33	...	...
239		25	25	2	10	151	317·1
240		Feb. 15	15	6	10	151	38·4
241	A	17	16	23	...	...	15·9
...	A	18	...		...	...	...
...		19	...		60	...	...
242		21	21	2	...	152	321·6
...		22	...		40	...	...
243		Mar. 15	15	7	9	...	29·0
244		20	20	3	15	153	325·3
245		Apr. 17	16	6	...	154	327·4
...		18	...		78	...	...
246		May 14	13	7	30	155	330·0
247	A	June 10	9	11	37	156	330·6
248	G	Aug. 2	2	6	...	158	338·7
...	G	3	...		...	...	...
...	A	4	...		...	...	...
...		5	...		83	...	...
249	A	10	10	6	13	...	232·9
250		14	14	7	...	...	179·5
...		1	...		28	...	...
251		Oct. 4	4	17	...	160	220·5
...	A	5	...		...	...	...
...		6	...		45	...	...
252	A	19	19	0	10	...	32·0
253		26	26	3	7	161	298·0
254		30	29	23	...	...	247·5
...	A	31	...		...	...	...
...		Nov. 1	...		82	...	...
255		3	3	5	7	...	191·4
		1866.					
256		Feb. 6	6	3	...	164	21·0
...	A	7	...		38	...	...
257	G	20	20	13	...	165	191·1
...	A	21	...		20	...	...
258	A	23	23	6	...	...	155·5
...		24	...		...	...	...
...	A	25	...		53	...	...

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1866.					
259		Mar. 7	6	8	32	...	9·5
260		18	18	7	19	166	211·9
261		Aug. 23	23	4	18	172	284·6
262		Sept. 9	9	5	8	...	59·5
263	A	Oct. 4	4	4	16	173	90·1
264		7	6	3	34	...	64·3
265		12	12	6	10	174	343·5
266		Nov. 26	26	3	12	175	111·7
		1867.					
267		Feb. 8	8	6	12	178	215·5
268		13	13	6	7	...	149·6
269		Mar. 6	6	5	...	179	233·5
...	A	7	...		...	...	...
...		8	...		55	...	...
270		10	10	6	9	...	180·3
271		May 28	28	7	11	182	216·4
272		June 1	1	11	10	...	161·3
273		Sept. 7	7	1	12	186	310·6
274		25	25	4	16	...	71·3
275		Oct. 2	2	?	18	187	337·3
		1868.					
276		Feb. 20	20	5	10	192	280·4
277		Mar. 20	19	15	32	193	266·0
278		30	30	9	9	...	124·2
279		Apr. 1	1	10	...	...	97·3
...		2	...		28	...	...
280		19	18	11	28	194	232·3
281	A	27	27	6	9	...	116·2
282		29	29	5	16	...	90·3
283		June 29	29	12	7	197	359·4
284	A	July 10	10	2	22	...	219·3
285		14	14	13	10	...	160·3
286	A	Aug. 30	30	6	17	199	262·7
287	A	Sept. 15	15	13	12	...	47·6
288		27	27	5	17	200	253·6
289	G	30	30	6	14	...	213·5
290		Oct. 19	19	4	11	201	323·9
291	A	22	22		20	...	284·9

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1868.				
292		Oct. 24	24 3	...	...	258·5
...		25	...	45	...	...
293		Nov. 19	19 4	12	202	275·1
		1869.				
294		Jan. 21	20 12	20	204	174·0
295		Feb. 2	2 11	...	...	3·4
...	A	3	...	...	...	...
...		4	...	53	...	...
296		Mar. 9	9 7	...	206	264·6
...	A	10	...	32	...	...
297		18	18 1	10	...	149·3
298	A	Apr. 2	2 5	12	207	309·3
299		8	8 5	13	...	230·1
300	G	15	14 23	...	...	141·0
...	A	16	...	42	...	...
301	G	May 13	13 2	12	208	129·3
302		30	30 18	4	209	255·6
303	A	June 6	6 14	...	...	165·2
...	A	7	...	17	...	...
304		29	29 7	...	210	224·6
...		30	...	32	...	...
305		July 3	3 10	8	...	169·9
306		18	18 9	10	211	332·1
307		Aug. 6	6 8	9	...	81·3
308		20	20 10	5	212	255·1
309		24	24 5	15	...	205·0
310		Sept. 3	2 9	40	...	83·9
311		5	5 7	8	...	45·4
312		9	9 11	12	213	350·4
313		11	11 9	11	...	325·1
314	A	13	13 11	...	...	297·5
...		14	...	27	...	...
315	A	27	27 6	...	...	115·5
...		28	...	...	...	...
...		29	...	57	...	...
316		Oct. 6	5 22	22	...	1·1
317		25	25 1	16	214	108·9
318		Jan. 3	2 14	...	217	272·4
...		4	...	41	...	...
319		30	29 10	31	218	279·0
320	G	32	31 23	20	...	245·6
321	A	Feb. 11	11 7	32	...	109·5
322		23	22 9	29	219	323·6
323	A	Mar. 21	21 12	22	220	326·2
324	A	Apr. 5	4 18	22	...	138·2
325		12	11 20	28	...	44·7
326		15	15 12	...	221	356·3
...		16	...	30	...	...
327		21	21 4	7	...	281·4
328		May 15	14 15	31	222	331·4
329	G	20	19 23	16	...	260·9
330		June 12	12 5	16	223	313·2
331	A	16	16 18	...	...	253·1
...		17	...	15	...	...
332	A	Aug. 19	19 11	...	225	130·2
...	A	20	...	33	...	...
333	A	Sept. 3	3 4	...	226	295·9
...		4	...	45	...	...
334	A	7	7 6	12	...	242·0
335		23	23 15	...	...	25·8
...	G	24	...	...	...	...
...	A	25	...	...	...	...
...	A	26	...	82	...	...
336		Oct. 1	1 3	6	227	286·8
337	A	14	14 4	12	...	114·7
338	G	24	23 22	...	228	346·1
...	G	25	...	45	...	...
339	A	Nov. 8	7 19	30	...	150·0
340		19	19 3	11	...	0·7
341		Dec. 16	15 16	...	229	10·8
...	A	17	...	49	...	...
		1871.				
342		Jan. 4	4 5	7	230	113·4
343		13	12 22	20	231	358·7

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1871.					
344	A	Feb. 11	10	11	...	232	342·9
...	G	12	...		...	...	...
...		13	...		71	...	...
345		26	26	4	14	...	136·1
346		Mar. 1	1	6	10	...	95·4
347		22	22	8	...	233	177·6
...	A	23	...		28	...	...
348		27	27	2	17	...	115·0
349	A	Apr. 1	1	8	16	...	45·7
350	G	9	9	5	...	234	301·8
...		10	...		28	...	...
351		13	13	9	12	...	246·8
352	A	17	17	7	...	...	195·1
...	A	18	...		31	...	...
353		28	28	1	...	...	53·0
...		29	...		50	...	...
354	G	June 17	17	12	14	236	105·6
355	A	July 21	21	13	...	237	15·0
...		22	...		35	...	...
356	A	Aug. 6	6	1	10	238	170·0
357		21	21	9	7	239	327·3
358	G	24	24	8	15	...	288·2
359		Sept. 7	7	8	8	...	103·3
360		Oct. 14	14	5	6	241	336·6
361	G	Nov. 2	1	6	34	...	98·7
362	A	9	9	7	...	242	352·7
...	G	10	...		34	...	...
363		19	19	19	36	...	214·2
		1872.					
364	G	Feb. 4	4	2	20	245	289·3
365		19	19	13	13	...	85·7
366		Mar. 1	1	14	17	246	300·3
367	A	Apr. 10	10	2	...	247	139·5
...	A	11	...		34	...	...
368		15	14	15	...	...	79·5
...	A	16	...		46	...	...
369	A	June 3	3	2	10	249	145·6
370	G	July 7	7	5	...	250	53·9

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1872.					
...	G	July 8	...		40	...	...
371		10	10	1	13	...	16·4
372		18	18	11	12	251	265·0
373		27	27	9	12	...	147·1
374	G	Aug. 3	3	4	...	...	57·2
...	A	4	...		48	...	...
375	A	8	8	7	16	252	349·5
376	A	14	14	6	24	...	270·7
377		25	25	7	8	...	124·8
378	A	Sept. 17	17	9	6	253	179·9
379		Oct. 5	5	11	14	254	301·3
380	G	14	14	10	...	...	183·1
...	A	15	...		...	...	...
...	A	16	...		...	...	...
...	G	17	...		...	...	...
...	A	18	...		96	...	...
381	A	Nov. 10	10	11	...	255	186·5
...		11	...		25	...	...
382		Dec. 9	9	8	12	256	166·0
383		14	14	4	9	...	102·3
384		17	17	1	13	...	64·4
		1873.					
385	A	Jan. 3	3	4	12	257	198·8
386	A	5	5	7	...	...	170·8
...		6	...		...	...	...
...	A	7	...		68	...	...
387		25	25	0	...	258	271·4
...		26	...		...	...	...
...		27	...		60	...	...
388		Feb. 8	8	7	...	...	83·2
...	A	9	...		31	...	...
389	A	Mar. 8	8	3	...	259	76·6
...	A	9	...		...	...	...
...	A	10	...		85	...	...
390		23	23	4	16	260	238·3
391		Apr. 1	1	7	...	...	118·0
...		2	...		37	...	...
392	A	18	18	4	...	261	255·2

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1873.					
...		Apr. 19	...		37	...	...
393		May 15	15	7	19	262	256·7
394		23	23	8	19	...	150·3
395		32	31	18	30	...	39·0
396		June 20	20	5	12	263	141·4
397	A	26	26	7	10	...	60·9
398		29	29	11	12	...	19·0
399		July 9	9	7	14	264	248·8
400		12	12	5	14	...	210·2
401		16	16	6	15	...	156·7
402		23	23	4	9	...	65·2
403		Aug. 5	5	9	12	265	250·5
404		Sept. 20	20	4	13	266	5·6
		1874.					
405	A	Jan. 15	15	8	...	271	261·0
...		16	...		...	...	...
...	A	17	...		72	...	...
406		27	27	6	16	...	104·1
407	G	Feb. 4	4	1	...	...	1·6
...		5	...		36	...	...
408	A	Mar. 7	7	5	28	273	311·1
409	A	Apr. 1	1	11	...	274	338·2
...	A	2	...		25	...	...
410	A	7	7	8	12	...	260·7
411		13	13	4	10	...	183·7
412		28	28	12	20	275	341·1
413		May 26	25	18	30	276	340·8
414		June 3	3	8	9	...	227·2
415		Sept. 29	29	19	10	280	101·0
416	G	Oct. 3	3	2	...	...	57·5
...	A	4	...		...	...	...
...		5	...		62	...	...
417		16	16	3	18	281	245·5
		1875.					
418		Feb. 26	25	23	...	286	295·4
...		27	...		28	...	...
419		Apr. 7	7	4	14	287	125·3
420		26	26	5	9	288	233·8
421		May 6	6	3	11	...	102·8
422		Sept. 15	15	12	15	293	152·0

Ref. No.	Class.	Days of Disturbance.	G.M.T. of Commencement of Disturbance. d	h	Duration. h	No. of Rotation.	Longitude of Sun's Centre. °
		1876.					
423	A	Feb. 19	18	16	33	299	253·8
424		Oct. 23	23	3	9	308	224·8
425		Nov. 10	10	8	6	309	344·7
		1877.					
426		Jan. 6	6	5	10	311	315·3
427		May 2	2	7	11	315	225·2
428		11	10	21	12	...	111·7
429	A	28	28	6	15	316	241·8
430		Oct. 11	11	12	13	321	240·6
		1878.					
431		Jan. 23	23	13	12	325	309·5
432	A	June 3	2	20	20	329	30·4
		1879.					
		None					
		1880.					
433		Mar. 17	17	5	7	353	47·6
434		May 2	2	3	17	355	161·4
435		Aug. 11	10	22	...	359	267·9
...	A	12	...		...	...	...
...	A	13	...		70	...	...
436		19	18	19	20	...	163·8
437		Sept. 14	14	13	...	360	170·5
...		15	...		27	...	...
438		27	27	11	10	...	0·0
439		Oct. 25	25	7	...	362	352·8
...		26	...		34	...	...
440	A	Nov. 3	2	17	21	...	241·8
441		Dec. 19	19	0	12	364	351·7
		1881.					
442		Jan. 21	21	15	11	365	268·9
443	G	31	30	21	19	...	147·1
444		Mar. 3	2	11	16	366	104·4
445		Apr. 20	20	9	12	368	179·2
446	G	Sept. 12	12	1	...	373	66·0
...	A	13	...		...	...	...
...		14	...		60	...	...
447		Nov. 8	8	1	...	375	34·0
...		9	...		40	...	...
448		23	23	6	18	376	193·5
449		29	28	23	16	...	118·2
450		Dec. 23	23	6	9	377	158·2

It will be seen from Table IX. that the magnetic disturbances of the thirty-five years 1848 to 1881 show just the same kind of tendency to recur at intervals of about twenty-seven days as do the disturbances of the twenty-two years 1882 to 1903, given in Table I. in my former paper. More than seventy "sequences" are shown in all; most being, as with the sequences of Table I., pairs, *i.e.* the disturbance returns once after an interval of one or two solar rotations, but is not observed again. But there are several instances of longer duration, and these are given in Table X. Sequence LIII. is especially interesting, since in this case the disturbance was observed in eight successive rotations, the times of the returns being exceedingly regular. One of these returns is not included in Mr. Ellis's catalogue, the movement on this occasion falling very slightly below his adopted standard. A series of disturbances such as this, so regular in the time of return and so long continued, is sufficient in itself to establish the contention of my earlier paper that our magnetic disturbances are due to an action arising from restricted areas of the Sun's surface, and reaching the Earth along restricted stream-lines, not by a general radiation; and the demonstration is the more complete since for much of the time that the series was observed no other disturbances were in evidence.

Some of the other sequences in Table X. have been completed by the insertion of a disturbance not in Mr. Ellis's catalogue.

Table X.

Reference No. of Sequence.	No. of Rotation.	Reference No. of Disturbance.	Longitude of Centre of Sun's Disc.	Reference No. of Sequence.	No. of Rotation.	Reference No. of Disturbance.	Longitude of Centre of Sun's Disc.
			°				°
LIII.	151	239	317·1	LV	−67	14	208·8
	152	242	321·6		−66	...	...
	153	244	325·3		−65	21	202·8
	154	245	327·4		−64	22	198·9
	155	246	330·0	LVI.	−24	55	72·2
	156	247	330·6		−23	56	84·3
	157	...	332·1		−22	...	...
	158	248	338·7		−21	61	100·3
LIV.	119	191	68·2	LVII.	0	83	3·7
	120	193	67·1		1	84	9·5
	121	194	70·5		2	86	7±
	122	195	77·4		3	88	16·1
	123	197	78·5				
	124	202	80·2				

Reference No. of Sequence.	No. of Rotation.	Reference No. of Disturbance.	Longitude of Centre of Sun's Disc.
			°
LVIII.	3	87	143·9
	4	89	152·2
	5	91	141·7
	6	94	156·6
LIX.	105	177	337·2
	106	...	...
	107	178	342·6
	108	180	343·2
LX.	141	220	319·6
	142	222	309·3
	143	224	293·2
	144	225	309·7
LXI.	146	229	161·9
	147	231	167·9
	148	233	173·0
	149	234	176·6
LXII.	149	236	31·1
	150	...	...
	151	240	38·4
	152	243	29·0
LXIII.	219	322	323·6
	220	323	326·2
	221	...	328·8
	222	328	331·4
LXIV.	−29	45	87·8
	−28	49	84·2
	−27	50	86·9
LXV.	131	209	119·4
	132	211	124·8
	133	213	114·2
LXVI.	146	227	234·5
	147	230	245·9
	148	232	225·1
LXVII.	199	286	262·7
	200	288	253·6
	201	292	258·5
LXVIII.	206	297	149·3
	207	300	141·0
	208	301	129·3
LXIX.	228	338	346·1
	229	340	360·7
	230	341	370·8

5. *Further Remarks on the Magnetic Disturbances*, 1882 *to* 1903.

I should like to be allowed to supplement the above new material which I have brought forward by a few remarks upon my earlier paper. First, with regard to its general scope and character. It was throughout in all its parts solely a discussion of observations. I advanced no "speculations," expressed no "theoretical views," ventured upon no "guess," favoured no "particular theory." I cannot find that there was any ambiguity in anything which I wrote in it, or in the two earlier papers which preceded it (*Monthly Notices*, vol. lxiv. No. 3, pp. 205, 222); but if there were such, let me make the matter clear now.

I dealt only with actual observations of magnetic storms and of sun-spots and of other solar phenomena. The result of this inquiry led to the discovery of a certain relation between the observed times of magnetic disturbances—namely, that they showed a tendency, too frequent to be accidental, to recur at intervals corresponding to one or other of the synodic rotation-

periods indicated by sun-spots. As a direct deduction from this observed relation, which for the sake of brevity and distinctness I will call the "Interval-Relation," it follows that there is a real connexion between the Sun and our disturbances; that the action of the Sun in that connexion is in each particular instance confined to a restricted area of the surface; and that the solar action is conveyed to us, not by radiation in all directions, but along restricted lines. I not only showed that the observed times of our magnetic disturbances indicated the existence of such stream-lines emanating from the Sun, but I showed that rays, restricted and defined, and thus analogous in form to those indicated by the disturbances, had been actually photographed as proceeding from restricted areas of the Sun. The existence, therefore, of such stream-lines is no more hypothetical or speculative than the existence of prominences, faculæ, or sun-spots themselves. In only one paragraph in my paper did I even refer to any speculative views, and then only for the purpose of illustration. In that case the speculation was not my own, and I was careful to preface my reference to it with the express disclaimer—"As to the physical cause of these streams and the condition of the matter composing them, it does not lie within my province to offer any suggestion" (*Monthly Notices*, vol. lxv. p. 33).

Nor did I express either directly or indirectly any opinion as to the source of the energy manifested in our magnetic storms.

It had been held—and high authority had been pleaded for the contention—that a direct connexion between the Sun and our magnetic disturbances was impossible unless we ascribed the energy manifested in them to an altogether impossible output of energy on the Sun. I showed that it followed simply from a consideration of the times at which the disturbances commenced that the solar action in them was altogether of a different kind from that which had been contemplated in this contention, and upon which the supposed output of energy had been calculated. I did not challenge the accuracy of the calculation, or in any way attempt to modify it; I simply showed that it had no bearing upon the case presented to us in nature. That calculation was irrelevant, and could not be taken into consideration. But beyond this I wrote no word bearing on the source of energy, whether it lay in the Sun, or in the Earth, or in some unknown region outside either. To remove the long "outstanding difficulty" it is sufficient that we now know that the actual method of action of the Sun is not that which had been contemplated, and upon which the calculation in question had been based.

I am able to write thus definitely with regard to the establishment of the chief contentions of my paper, since I find that there is a wide consensus of opinion that the Interval-Relation has been demonstrated, and it is no longer necessary for me to labour the point of its validity as if it were still in dispute. Professor Schuster says that an examination of my paper has convinced him "that, subject to certain qualifications,

Mr. Maunder has made good his contention that magnetic storms are apt to recur in periods not differing much from that of the mean rotation of the sun-spot zones." Again, referring to the same relation, he says: "He [Mr. Maunder] has, no doubt, added a new fact" (*Monthly Notices*, vol. lxv. pp. 186, 197). The Rev. A. L. Cortie in a paper which is in the main unfavourable to my conclusions, admits that "sequences undoubtedly exist" (*Monthly Notices*, vol. lxv. p. 203). Professor Larmor said that my paper "amply satisfied" him "that these magnetic storms somehow or other occur in a period which is nearly the same as the rotation of the Sun"; and, referring to Plate I., fig. 4, of my paper, added: "An inspection of the diagram throws more light on the matter than all the tables that could be invented" (*Observatory*, 1905 February, pp. 84, 85).

It is granted then, that, as Professor Schuster puts it, "we may accept it as proved that magnetic storms show some kind of periodicity, the length of the period being somewhere near 27·27 days" (*Monthly Notices*, vol. lxv. p. 189). There are only two bodies of which we know which could give rise to an interval of the kind—the Sun and the Moon. In the case of the Moon we have the three months: the draconitic of 27·2122 days, the sidereal of 27·3217 days, and the anomalistic of 27·5546 days. A glance at the following table, in which the sequences of my first paper are given together with their corresponding intervals, shows that my results are not consistent with the view that the acting body is the Moon in one or other of the three above-mentioned periods, since in so many instances the period involved is widely different from any of the lunar periods, and the disturbances fall without discrimination at all times and epochs of the various months concerned.

There is left, therefore, only the Sun, and here the wide range of periods indicated is not a difficulty but a corroboration; for such a range of motion is precisely what the sun-spots exhibit to us. And it should be noted here that though it is true that there is a gradual lengthening of the *mean* rotation-period for each latitude as we recede from the equator, yet there is so wide a range of motion in any particular latitude that all of the rates of motion indicated in Table X. might belong to spots anywhere between 5° and 30° of latitude; the great majority of them to spots anywhere between 0° and 35°.

TABLE XI.

Sequences and Corresponding Rotation-periods.

Reference No. of Sequence.	No. of Terms.	Apparent Drift in Longitude in a Rotation.	Apparent Daily Drift in Longitude.	Mean Synodic Rotation Period.	Daily Sidereal Motion.	Sidereal Rotation Period.
		°	′	d	′	d
XXVII.	3	+ 9·6	+ 21·1	26·566	872·2	24·764
XXXV.	4	+ 9·1	+ 20·0	26·603	871·1	24·796
LXXII.	5	+ 8·9	+ 19·6	26·615	870·7	24·808

Reference No. of Sequence.	No. of Terms.	Apparent Drift in Longitude in a Rotation.	Apparent Daily Drift in Longitude.	Mean Synodic Rotation Period.	Daily Sidereal Motion.	Sidereal Rotation Period.
		°	′	d	′	d
XXXIII.	2	+ 6·3	+ 13·9	26·806	864·9	24·972
XXV.	4	+ 4·3	+ 9·5	26·953	860·5	25·101
XLVIII.	3	+ 4·3	+ 9·5	26·953	860·5	25·101
III.	2	+ 4·0	+ 8·8	26·975	859·9	25·119
I.	2	+ 3·9	+ 8·6	26·979	859·7	25·126
XLIV.	2	+ 3·7	+ 8·1	26·998	859·2	25·139
XX.	2	+ 3·5	+ 7·7	27·012	858·8	25·152
XVIII.*	4	+ 3·2	+ 7·0	27·034	858·1	25·172
XXXIX.	2	+ 2·8	+ 6·2	27·064	857·2	25·197
XV.	2	+ 2·7	+ 5·9	27·072	857·0	25·204
LII.	2	+ 2·05	+ 4·5	27·121	855·6	25·246
XL.	2	+ 1·9	+ 4·2	27·131	855·3	25·255
XIV.	2	+ 1·7	+ 3·7	27·147	854·8	25·268
VIII.	2	+ 1·4	+ 3·1	27·169	854·2	25·288
XLIII.	2	+ 1·25	+ 2·8	27·181	853·8	25·298
IV.	2	+ 1·2	+ 2·6	27·184	853·7	25·300
XLVII.	2	+ 0·9	+ 2·0	27·206	853·1	25·321
XXXVII.	2	+ 0·65	+ 1·4	27·226	852·5	25·337
LXXI.	6	+ 0·2	+ 0·4	27·260	851·5	25·367
XLIX.	2	+ 0·05	+ 0·1	27 271	851·2	25·378
XXIII.	4	− 0·2	− 0·4	27·290	850·6	25·392
XXVIII.	2	− 0·2	− 0·4	27·290	850·6	25·392
XLV.	2	− 0·3	− 0·7	27·298	850·4	25·399
IX.	2	− 0·6	− 1·3	27·320	849·8	25·418
LI.	3	− 1·1	− 2·4	27·358	848·7	25·452
XXXIV.	2	− 1·4	− 3·1	27·381	848·0	25·472
XXXI.	3	− 1·7	− 3·7	27·404	847·3	25·491
XXI.	4	− 1·8	− 4·0	27·412	847·1	25·498
VII.	2	− 2·6	− 5·7	27·473	845·4	25·551
XLII.	2	− 2·7	− 5·9	27·481	845·1	25·558
LXX.	7	− 3·0	− 6·6	27·504	844·5	25·578
XXXVIII.	2	− 3·25	− 7·2	27·558	843·9	25·595
II.	3	− 4·6	− 10·1	27·628	841·0	25·688
XI.	2	− 5·0	− 11·0	27·659	840·1	25·712
XIII.	2	− 6·0	− 13·2	27·737	837·9	25·779

* Increased to four terms by the inclusion of Disturbance No. 250.

Reference No. of Sequence.	No. of Terms.	Apparent Drift in Longitude in a Rotation.	Apparent Daily Drift in Longitude.	Mean Synodic Rotation Period.	Daily Sidereal Motion.	Sidereal Rotation Period.
		°	′	d	′	d
XLVI.	2	− 7·4	−16·3	27·847	834·8	25·874
V.	2	− 7·5	−16·5	27·855	834·6	25·881
XXXVI.	2	− 8·2	−18·0	27·911	833·0	25·929
XXII.	2	− 8·5	−18·7	27·934	832·4	25·949
XXX.	2	− 9·7	−21·3	28·031	829·7	26·032
L.	2	−10·15	−22·3	28·066	828·7	26·064
X.	2	−11·0	−24·2	28·135	826·9	26·122
VI.	2	−12·6	−27·7	28·264	823·4	26·234
XXIV.	2	−15·8	−34·8	28·565	816·3	26·460
XXIX.	3	−16·5	−36·3	28·585	814·8	26·510

In the above table I have included three sequences not given as such in my former paper. I give them here as illustrations of the way in which I might have legitimately extended the number of sequences and their completeness if I had not preferred to retain the catalogue, which I had drawn up at the commencement of my inquiry, absolutely unaltered. It was, as I explained, based simply upon the disturbances which were illustrated in the plates of the Greenwich volumes. Since these plates are given where the disturbance does not lend itself conveniently to description it follows that not a few disturbances of well-marked character and considerable amplitude are unrepresented, inasmuch as they have a sufficiently full description in the "Notes." If these are added to our catalogue then a number of sequences are formed of a higher order than those which I gave. The three in question may serve as examples of these. Sequence LXX. is formed from Sequences XVI. and XVII. by the inclusion of a new and important but simple disturbance ; and Sequence LXXI. from Sequences XII. and XLI. ; Sequence LXXII. from Sequence XIX. by similar additions.

TABLE XII.

Ref. No. of Sequence.	Ref. No. of Disturbance in Table I.	Class.	Greenwich Civil Time of Commencement.	d	h	No. of Rotation.	Longitude of Sun's Centre.
							°
LXX.	205	A	1895 Oct.	12	16	562	222·8
	206	A	Nov.	9	13	563	215·2
	...	M	Dec.	7	7	564	209·6
	207	A	1896 Jan.	3	0	565	217·7
	208	M		31	12	566	202·4
	210	V	Feb.	28	9	567	195·4
	212	M	Mar.	26	13	568	197·2

Ref. No. of Sequence.	Ref. No. of Disturbance in Table I.	Class.	Greenwich Civil Time of Commencement.			No. of Rotation.	Longitude of Sun's Centre.
				d	h		°
LXXI.	169	A	1893 June	18	13	531	232·7
	170	V	July	15	22	532	230·4
	...	M	Aug.	12	6	533	228·9
	174	M	Sept.	18	1	534	235·0
	...	M	Oct.	5	13	535	232·0
	177	V	Nov.	1	15	536	234·7
LXXII.	183	V	1894 Feb.	22	23	540	182·3
	186	M	Mar.	21	12	541	192·1
	188	V	Apr.	17	13	542	195·2
	...	M	May	14	19	543	216·0
	189	A	June	9	14	544	213·8

6. *The Distribution-Relation in Sun-spots.*

In my earlier paper I drew attention to a relation of the magnetic disturbances other than that which I have called the "Interval-Relation." For the sake of distinctness I will call this second relation the "Distribution-Relation." It was indeed the first point to which I called attention in referring to my diagram, fig. 4. ("Distribution of Magnetic Disturbances," *ibid.* vol. lxv. Plate 1.) I wrote (*Monthly Notices*, p. 18), "A mere inspection of the diagram brings out a striking and most important relation. The disturbances are not distributed irregularly with regard to the solar meridians, but chiefly affect one or two regions." I may say that to myself this was the most interesting result of my inquiry, although I considered that I was bound to defer enlarging upon it until I had demonstrated the Interval-Relation and the consequences arising from it. For this Distribution-Relation is distinct from that which I have termed the Interval-Relation, and is no necessary consequence of it. The latter I consider that I have now established; the Distribution-Relation, I think, is strongly suggested by my diagram, but I would by no means claim that it is proven.

I was therefore exceedingly gratified when Professor Larmor fixed upon this point as "a fact of extreme importance, even of more importance than the periodicity" (*Observatory*, 1905 February, p. 85). It was a further gratification to me that he also suggested that "it would be a very interesting thing if Mr. Maunder could show us diagrams like that one, each diagram classifying the sun-spots which occur in that narrow zone of latitude where the period of revolution shown by the sun-spots was something fairly definite," for that was precisely the work upon which my wife and I were employed when I came across the Interval-Relation.

As a very limited and partial example of the work upon which we have been engaged, we beg to submit the accompanying

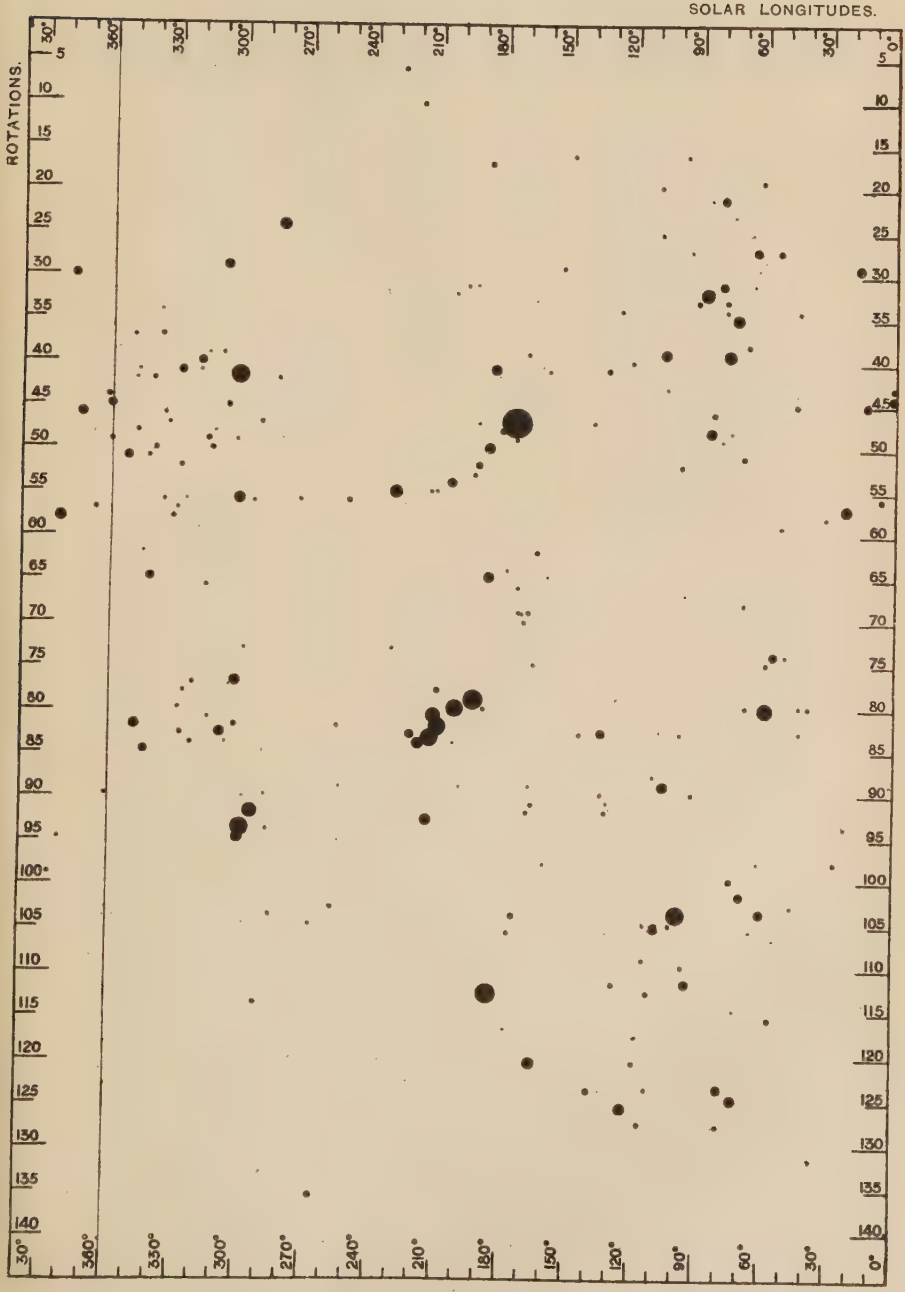
diagram. It shows all the spot groups, small and great, ephemeral and long-lived, given in the Greenwich Heliographic Results between the equator and north latitude 7·5 for the whole of the cycle 1890–1902. The first group shown was observed 1891 August 8, the last 1901 July 8. There is no possibility of doubt between one cycle and the next when limited zones in latitude are considered ; they are marked off from each other by broad barren intervals of time, within which no spots whatever are seen in that particular zone. The last spot of the preceding cycle was observed twenty-one rotations earlier, on 1890 January 6. The first spot of the new cycle, now in progress, was seen in this zone on 1904 April 11.

When the sun-spots of the different zones of latitude for the cycle 1889–1901 were charted down according to their longitudes as reduced by Carrington's sidereal period of 25·38 days, corresponding to a mean synodic period of 27·275 days (the rotation-period adopted at Greenwich), two narrow but well-marked barren belts were observed to run in a slanting direction across the diagram. We have therefore recomputed the mean longitudes for all these spot-groups with an assumed sidereal period of 25·09 days, corresponding to a synodic period of 26·94 days. This period is probably open to some slight correction, and it must be perfectly borne in mind that any division of spot-groups by a hard-and-fast line of latitude must inevitably include a considerable number of outlying eruptions really belonging to adjacent zones, and therefore moving at different speeds. Further, it must not be forgotten that, though we may determine a mean rotation-period for each zone, that does not in the least predicate an absolutely uniform rotation-period for that zone. On the contrary, the internal motions in any given zone, and even in any single spot-group of long duration, are very considerable. The net result of all these is greatly to mask the existence of any real barren regions, a masking effect which is still further accentuated by the tendency of great eruptions to extend themselves in longitude over long arcs. Yet a glance at the diagram shows that all these effects have not sufficed completely to efface the division of this very considerably wide zone into three great regions of action varying in their activity in four successive epochs.

TABLE XIII.

Distribution of Total Spotted Area between 0° *and* +7°·5.

Solar Longitude.		Percentages of Total Areas.				
From	To	1891 Aug. 8 to 1895 July 17.	1895 Sept. 15 to 1896 Sept. 18.	1896 Oct. 18 to 1898 Feb. 26.	1898 May 22 to 1901 July 8.	Entire Cycle.
353°	8°	4·1	...	...	...	1·8
8	23	5·7	...	...	...	2·4
23	38	...	...	...	0·3	...
38	53	0·7	0·7	0·2	0·1	0·4



NORTH EQUATORIAL SPOTS, 0° TO + 7°.5;

SHOWING THEIR DISTRIBUTION IN SOLAR LONGITUDE FOR THE CYCLE, 1891 TO 1902. SIDEREAL ROTATION PERIOD ADOPTED, 25.09 DAYS. DATE OF COMMENCEMENT OF ROTATION NO. 1: 1891 MARCH $13^{d}.76$, GREENWICH CIVIL TIME.

Solar Longitude. From	To	Percentages of Total Areas. 1891 Aug. 8 to 1895 July 17.	1895 Sept. 15 to 1896 Sept. 18.	1896 Oct. 18 to 1898 Feb. 26.	1898 May 22 to 1901 July 8.	Entire Cycle.
53°	68°	2·2	**22·9**	**6·8**	6·6	4·8
68	83	**12·6**	0·6	...	15·3	**8·0**
83	98	6·3	...	...	2·7	3·2
98	113	2·9	...	3·4	**24·1**	6·6
113	128	0·2	...	...	10·3	1·8
128	143	0·5	...	1·6	2·1	1·1
143	158	...	...	...	...	...
158	173	...	6·0	0·4	6·6	1·5
173	188	**35·3**	**49·4**	...	**29·8**	**22·1**
188	203	0·6	...	16·0	...	6·2
203	218	1·3	...	**35·6**	...	**13·4**
218	233	4·8	0·2	1·2	...	2·5
233	248	...	...	...	...	...
248	263	...	...	...	0·3	0·1
263	278	...	...	...	1·3	0·2
278	293	2·6	...	...	0·3	1·2
293	308	**10·9**	0·2	**25·1**	...	**14·0**
308	323	2·9	1·6	3·8	...	2·7
323	338	4·0	...	0·4	...	1·9
338	353	0·7	**18·1**	4·9	...	3·0

The first epoch begins with the commencement of the cycle 1891 August 8 ; but the spots are very sparse at first, and it is not until nearly a year later that the activity of the zone may be said to have really commenced. The epoch lasts until 1895 July 17, and is on the whole the most active of the four. The second epoch is one of almost complete quiescence, and lasts for a little over a year, its first spot being seen 1895 September 15, its last 1896 September 18. The third epoch is again one of great activity, extending from 1896 October 18 to 1898 February 23. The fourth and last epoch is one of decadence : its first spot was observed 1898 May 22, its last 1901 July 8.

Throughout all four of these epochs the three great regions of activity are clearly to be distinguished, but they are especially marked in the first and third. In the second epoch almost all the spots are contained in a narrow region extending over only 30° of longitude, and corresponding to the most active of the three centres. In the fourth epoch, for a period of very nearly three years, the entire spot-activity of the Sun is confined within 130° of longitude, leaving the remaining 230° almost absolutely void. This remarkable inequality seems to be due to the entire

subsidence of the second great region of activity and the marked diminution of the first centre, leaving the third in almost exclusive possession of the field.

These three great centres of activity are marked off from each other by broad barren regions. The broadest of these is nearly 60° wide, lying between the first and second centres of activity, and with its centre about 260° of longitude. The second barren region lies between the third and the first centres, and is about 45° wide, with its centre of longitude 140°. The third and narrowest lies between the second and third centres, and is about 25° wide, with its centre of longitude 40°. The three centres of activity have their positions, the first around longitude 200°, the second around longitude 340°, and the third around longitude 80°.

The diagram shows *all* the spots within the latitudes named for a period of fourteen years and three months—191 rotations of Carrington's period. During this interval of time a spot moving with the rotation-period adopted would gain two rotations and one-third upon another moving at Carrington's rate. A slightly smaller value for the rotation-period, one gaining two rotations and a half in the interval, would have satisfied the observations even better.

The great barren belt at longitude 260° can be traced, though with ever-increasing difficulty, right up to latitude 12°. If the present chart had been restricted to the spots between 6° and the equator this belt would have been broader and more sharply defined than it is, for there seem to be clear traces of the intrusion here and there of outliers from centres of activity in higher latitudes; but we have been careful in the present diagram to give *every* spot-group within the limits of time and of latitude chosen, no matter whether there was sufficient evidence to connect it with another zone or not.

The above remarks apply to but a single zone in one cycle, and we hope ere long to be able to follow it by a much fuller discussion of all zones, both north and south, for two complete cycles of the Sun. Bearing in mind how much there is still to do, we beg to offer this first rough instalment with all the reserve that the incomplete state of the work requires. We offer it as illustration only, not as demonstration. But the trend of our own thought during the last two years has been largely in the direction of the closeness of the analogy between what is so familiar to us in the behaviour of the markings on *Jupiter* and the behaviour of centres of spot-activity on the Sun. We have seemed—at least to ourselves—to recognise evidence of what, for want of a better term, we may perhaps term "structure" on the Sun—the setting-up over a wide region of the surface of a condition favourable to spot-formation, and possibly to that action the evidence of which we see in the disturbances of our terrestrial magnetism. Such regions are surrounded by others which appear almost wholly incapable of spot-formation,

and thus a differentiation is set up which may last for even a term of years. These regions of activity may show many changes in its manifestation during that time. The activity may be intermittent; it may oscillate from one latitude to another; it may vibrate backwards and forwards in longitude; and yet its continuity may not be wholly lost. The analogy of the Great Red Spot of *Jupiter* may help us here. It has been continuously under observation for nearly thirty years; very possibly it has existed for more than seven times as long. But taking the shorter period, it has during that time varied much in colour and intensity, slightly in size and shape; it has been very intermittent in its distinctness; its period of rotation has differed much from those of neighbouring spots, and has varied from one year to another. Yet it has preserved its individuality through all, and there is no doubt of the continuity of its existence. Though it is evidently no solid body—for its changes of motion preclude that idea—yet it evidently denotes something of structure, something of specialisation, in that region of *Jupiter*. So though we cannot predicate—or certainly not as yet—any such long-continued specialisation of any regions upon the Sun, yet we think that very strong suggestions have been afforded us that a certain degree of specialisation, though of a less permanent character, is already in progress there.

86 *Tyrwhitt Road, St. John's, Brockley, S.E.:*
1905 *March* 6.

Further Note on Instrumental Errors affecting Observations of the Moon: in reply to Mr. Cowell's Paper of 1904 *June.* By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

Owing to my absence in America, Mr. Cowell's paper of June last did not come to my notice in the ordinary course, and was only read by me comparatively recently. Though I am reluctant to prolong a controversy of little general interest, there are at least two statements in the paper which seem to me to require important corrections, and I hope to indicate them as briefly as possible. The first statement is as follows (bottom of p. 695):

"Professor Turner . . . has thrown aside the observations of what he calls 'day 15' (or full moon) and smoothed out days 14 and 16. These are the most important observations of all, because the parallactic inequality is then most different from its mean value, and we are at once left with an amount of material insufficient for its purpose."

This statement is irrelevant, because I was not dealing with the parallactic inequality or concerned with it in any way whatever. I was discussing *instrumental error*, the motion of the

Moon with all its characteristics having been eliminated by the simple process of comparing observations made by one instrument with those made by another at virtually the same moment. The observations might have been made, for instance, on a fixed artificial disc. My procedure cannot therefore be criticised from the point of view applicable in the case of the parallactic inequality.

I have pointed this out to Mr. Cowell, but he still maintains that his criticism applies because my discussion of instrumental error suggested a word of caution as to the correctness of his value for the parallactic inequality. The point is rather a logical than a mathematical one, and I will venture to illustrate my contention of irrelevance by an analogy.

Suppose Mr. Cowell to be discussing the chances of a broken leg, to which travellers are liable, from statistics of journeys between London and Liverpool, and that, in order to be strictly just to the railways, he acknowledged that some of the accidents might have occurred at home, setting down, without indicating how he arrived at it, an estimate of that number. Suppose, further, that from the history of stay-at-homes who take no journeys * I arrived at the conclusion that the estimate so made was inadequate; then surely my methods for discussing stay-at-homes are not to be judged by the rules applicable to travellers simply because the conclusion derived from one case happens to have a bearing on the other. To be more specific, suppose I found it advisable to neglect the hours when the stay-at-homes were in bed as not seriously liable to accident (this is taken to correspond to "throwing aside day 15 &c."). Mr. Cowell's attitude seems to me equivalent to stating that "these hours are the most important of all, because *in railway travelling* accidents are most frequent at night." In essentials this seems to me a fair analogy. Mr. Cowell's eyes are fixed so intently on the parallactic inequality that he treats instrumental error as a particular case of it, whereas the form of the latter is quite different and quite arbitrary. It would be as logical to treat a stay-at-home as a particular case of a traveller who made a return journey in no-time, and to insist on his taking out two insurance tickets accordingly.

The second point is with regard to $\sin D$ and $\cos D$ † on p. 696. Here we have been more fortunate in arriving at an understanding if not an agreement. The question whether a quantity is to change sign at full moon is liable to ambiguity

* My analogy is of course imperfect here, in that had such direct evidence been available Mr. Cowell would naturally have used it. The comparison of two instruments which I made gives no direct evidence of the instrumental error of either; but at least the motion of the Moon (corresponding to the journey) is eliminated, which is the essential point.

† I must apologise for having inadvertently measured D from full moon instead of new moon on p. 568 of vol. lxiv., as Mr. Cowell indicates on p. 696.

because the longitude of the centre is made up of two terms—

$$\underset{\text{longitude of limb}}{\text{P}} \pm \underset{\text{observed semidiameter}}{\text{Q}}$$

of which P does not, and Q does, automatically change sign at full moon. Errors in longitude of the Moon naturally associate themselves with P, and were chiefly present to Mr. Cowell's mind; instrumental errors, especially the variability in observed semidiameter with approaching daylight, equally naturally associate themselves with Q, and were chiefly present to my mind (as may be seen on reference to *Monthly Notices*, vol. lxiv., bottom of p. 410). Hence it is clear that we cannot decide the applicability or non-applicability of terms such as sin D or cos D by their changing sign at full moon; we merely determine whether they are associated with P or Q.

Thus in the series suggested by me on p. 567

$$a_1 \sin \text{D} + b_1 \cos \text{D} + a_2 \sin 2\text{D} + b_2 \cos 2\text{D} + \&\text{c.}$$

for representing instrumental error, if we are thinking of Q we should expect the part

$$\text{C} = b_1 \cos \text{D} + b_2 \cos 2\text{D} + \&\text{c.}$$

to be large and

$$\text{S} = a_1 \sin \text{D} + a_2 \sin 2\text{D} + \&\text{c.}$$

to be small; if we are thinking of P being affected it might be the other way. Hence I do not see that Mr. Cowell's paragraph on p. 696 (when the above possible misunderstanding is cleared away) is any answer to the remark I made on p. 568. For clearness may I recall very briefly the sequence of events?

I raised the question whether the instrumental errors of the old transit circle in use to 1851 were not sensibly different from those of the new one used afterwards.

Mr. Dyson replied (p. 566) that the coefficients of sin D, deduced by Mr. Cowell from the observations, showed no such change.

I rejoined (p. 567) that such evidence was incomplete, since terms involving cos D, cos 2D, &c., had not been included in the series, and that these had probably a very real existence for reasons given.

To this I venture to think no real response has been made either on p. 696 or otherwise. It is true that Mr. Cowell has since looked for, and found, a term $0''{\cdot}28 \cos \text{D}$ (p. 695, line 9) "which must be attributed to errors of observation"; but I believe this is applicable to longitude (called P above) and not to semidiameter (Q above). It seems to me, in the light of our recent discussion, that a complete investigation would contemplate *four* sets of terms: viz. (1) sines applicable to P, (2) sines

applicable to Q, (3) cosines applicable to P, (4) cosines applicable to Q. But in practice it would be doubtless impossible to separate these terms satisfactorily.

Before leaving the point perhaps I may state my position in other words thus :

Mr. Cowell writes (vol. lxiv. p. 96) : "The hypothesis that the error of semidiameter is a constant is the best that I can make."

I venture to doubt this, even in the circumstances considered by Mr. Cowell. I think he should have considered the expression

$$b_0 + b_1 \cos D + b_2 \cos 2D + \&c.$$

instead of the first term only.

The remainder of Mr. Cowell's paper is an admirable exposition of the principles of his analysis, which I am glad to have drawn from him. If I refrain from comment it is because I am afraid of prolonging the discussion unduly. The two points mentioned seem to me important, and I hope I have made my position with regard to them clear. Mr. Cowell and I have, as already mentioned, discussed these matters without being able to arrive at an agreement, so that he will doubtless represent his views clearly also, and there I hope the matter may be left.

Reply to Professor Turner's further Note. By P. H. Cowell.

1. Let me put

N = instrumental error of transit circle (1851–61)
O = „ „ "old transit circle" (1847–50)
L = „ „ altazimuth (1851–61)
M = „ „ „ (1847–50)

Then Professor Turner has shown, by processes that I have no objection to, that N—O—L+M varies from $0''.0$ on day 17 to $+1''.3$ on day 21, and then decreases to $+0''.8$ on day 24, the zero being an arbitrary one (vol. lxiv. p. 411).

Regarding this table merely as proving that the above combination of instrumental error varies with the day of the lunation, it was clearly unnecessary for me to call attention to the absence of days 15 and 16. The importance of these days is, however, considerable when we attempt to consider the law connecting the above observed quantity with the age of the Moon.

I do not wish my chief objections to Professor Turner's first paper to be obscured by a discussion of minor points. What I object to is this. He says L—M may be put equal to zero, and

he puts N—O instead of N—O—L+M at the top of the table referred to. From some analyses of my own, given on vol. lxiv. p. 580, it may be inferred that the values of N—O are really about one-third of those given by Professor Turner. The other two-thirds must apparently be ascribed to L—M, and give some indication of the accidental errors to which the altazimuth was liable.

2. The formula to which I equate the errors is

$$\begin{aligned}\varepsilon \pm \mu + \delta_1 \sin D + \delta_2 \sin 2D + \ldots \\ + \Delta_1 \cos D + \Delta_2 \cos 2D + \ldots\end{aligned}$$

The formula used by Airy (*Memoirs* R.A.S., vol. xxix. p. 4) is

$$\delta_1 \sin D + \delta_2 \sin 2D$$

Professor Turner suggests

$$\begin{aligned}&\pm(\mu + b_1 \cos D + b_2 \cos 2D + \ldots \\ &\quad + a_1 \sin D + a_2 \sin 2D + \ldots) \\ &+ \varepsilon + \delta_1 \sin D + \delta_2 \sin 2D + \ldots \\ &+ \quad \Delta_1 \cos D + \Delta_2 \cos 2D + \ldots\end{aligned}$$

My formula is therefore intermediate between that used by Airy and that suggested by Professor Turner.

There is a phrase, "necessary and sufficient," in constant use by mathematicians. I maintain that Airy's formula is insufficient and Professor Turner's unnecessary. Airy's is insufficient because the deduced value of the parallactic inequality is clearly erroneous if his semidiameter is erroneous. Airy has, in effect, therefore made his parallactic inequality depend, not upon the 10,000 observations that he reduced, but upon the 100 observations that he used for semidiameter.

Each one of the additional terms proposed by Professor Turner can, on the other hand, be shown to be unnecessary. I will take one only as an example, $\pm b_1 \cos D$. This may be replaced by

$$b_1(\pm 1{\cdot}000 - 1{\cdot}056 \sin D + 0{\cdot}478 \sin 2D)$$

for the expression that I have just written down differs from $\pm b_1 \cos D$ by considerably less than $\frac{1}{10} b_1$ from $D = 70°$ to $D = 290°$.

A similar remark being true of all Professor Turner's other terms as well, it is clear that he adds nothing to the generality of my hypothesis by assuming that the semidiameter is

$$\begin{aligned}&\mu + b_1 \cos D + b_2 \cos 2D \\ &\quad + a_1 \sin D + a_2 \sin 2D\end{aligned}$$

rather than by assuming that it is

$$\mu \pm (a_1 \sin D + a_2 \sin 2D + \beta_1 \cos D + \beta_2 \cos 2D)$$

in which case the inequalities of semidiameter merge into the inequalities of longitude.

My point can also be demonstrated thus. Remove $\pm\mu$ from my formula, and it becomes the most general expression possible for errors periodic in the lunation and free from discontinuity. The proper provision to make for a single discontinuity is to introduce a single additional unknown quantity.

The Coefficient of the Principal Term in the Moon's Latitude.
By P. H. Cowell.

1. The coefficient of sin F in Hansen's tabular latitude transformed to Delaunay's form is	18461″·65
2. The mean value of the observed *minus* tabular errors of 5646 observations from 1847 to 1901, each multiplied by 2 sin F, is... ...	−0″·27±0″·03
3. The correction to the deduced coefficient, on account of the excess +0″·11 sin (F−D) −0″·05 sin (F+D) of Hansen's tables over Brown's coefficients (*Monthly Notices*, vol. lxv. 1905 January), is	−0″·03
	18461″·35±0″·03
4. Estimated correction to reduce to Pulkowa refractions...	0″·00±0″·02
5. Estimated effect of planetary terms of period nearly equal to that of F	0″·00±0″·04
Concluded coefficient	18461″·35±0″·05

In paragraph 2 the accidental error is estimated from the analogous investigations for the longitude.

In paragraph 4 account is taken of the fact that Stone's refractions were used during the years 1868–1877, in which the longitude of the node decreased from 150° to −50°.

In paragraph 5, as the planetary perturbations in latitude have not yet been computed, an estimate of their possible effect was based upon the fact that the maximum effect of the planetary terms upon the principal elliptic inequality as deduced from the observations of any continuous period of fifty-four years is 0″·10.

Note on Point Distributions on a Sphere, with some Remarks on the Determination of the Apex of the Sun's Motion. By H. C. Plummer, M.A.

1. In several astronomical problems we have to consider the distribution on a sphere of a large number of points which do not appear to be scattered uniformly, but, on the contrary, reveal a tendency, more or less pronounced, towards some great circle on the sphere. An obvious instance is the distribution of the stars in the sky, and the question has been discussed from this point of view by Professor Newcomb in his recent paper "On the Position of the Galactic and other Principal Planes toward which the Stars tend to crowd." * The determination of the position of such a great circle or "plane of condensation" is required in other connexions, the instance quoted being the simplest and most definite illustration.

The stars are represented by n points on a sphere of unit radius. They are supposed to be crowded in the vicinity of a certain great circle, but the divergences cannot be regarded in general as small, and the law of crowding is unknown. Under these circumstances it is necessary to define the plane of condensation, and Professor Newcomb defines it as the plane of that great circle for which the sum of the squares of the sines of the distances of all the stars is a minimum. In other words, if v is the distance of any star from the pole of the great circle, the plane of condensation is such that

$$\Sigma \cos^2 v \text{ is a minimum}$$

Now let a coordinate system of rectangular axes $O\xi\eta\zeta$ be taken at the centre of the sphere, $O\xi$ passing through the pole of the required circle. Then the preceding condition becomes

$$\Sigma\xi^2 \text{ is a minimum}$$

But $$\xi^2+\eta^2+\zeta^2=1$$

Hence $$\Sigma(\eta^2+\zeta^2) \text{ is a maximum}$$

If then we assign unit mass to each of the points on the sphere, the condition in its last form shows that the moment of inertia of the system about $O\xi$ is the greatest possible. Hence the problem is reduced simply to that of finding one of the principal axes of inertia for the system of loaded points on the sphere.

With any system of rectangular axes $Oxyz$ (*e.g.* the equatorial system) let (a, b, c) be the coordinates of any one of the stars, and let

$$A = \Sigma a^2,\ B = \Sigma b^2,\ C = \Sigma c^2,\ F = \Sigma bc,\ G = \Sigma ca,\ H = \Sigma ab$$

* Contributions to Stellar Statistics. First Paper. Washington (1904).

The momental ellipsoid can be written

$$(B+C)x^2+(C+A)y^2+(A+B)z^2-2Fyz-2Gzx-2Hxy=1$$

which can be transformed by a change of axes to *

$$\mu_1\xi^2+\mu_2\eta^2+\mu_3\zeta^2=1$$

where (μ_1, μ_2, μ_3) are the roots of the equation

$$\begin{vmatrix} B+C-\mu & -H & -G \\ -H & C+A-\mu & -F \\ -G & -F & A+B-\mu \end{vmatrix} = 0$$

Hence $O\xi$ is the axis required if μ_1 is the greatest root of this equation.

This result differs in form from that found by Professor Newcomb, but the latter can be deduced immediately. For let

$$\lambda+\mu=A+B+C$$

Then the value of λ corresponding to μ_1 is clearly the smallest root of the equation

$$\begin{vmatrix} A-\lambda & H & G \\ H & B-\lambda & F \\ G & F & C-\lambda \end{vmatrix} = 0$$

and the associated system of equations which gives the position of the axis $O\xi$ relative to the axes $Oxyz$ is

$$\begin{aligned} (A-\lambda)x+ Hy + Gz &= 0 \\ Hx+(B-\lambda)y+ Fz &= 0 \\ Gx+ Fy +(C-\lambda)z &= 0 \end{aligned}$$

The plane $O\eta\zeta$ is the plane of condensation and is thus determined in a way formally the same as Professor Newcomb's. The dynamical analogy appears interesting in itself and has the advantage that questions of invariance and of the relative positions and magnitudes of the axes are answered by the simplest geometrical considerations. Professor Newcomb has already found some interesting results by the use of this method, and its more extensive application may be expected to yield still more important conclusions.

2. The second problem which reduces to the same fundamental question is the determination of the direction of the Sun's motion in space by Bessel's method. This is the method which has been employed by Dr. Kobold.† The form of solution which involves the discriminating determinant was pointed out by

* Cf. *Monthly Notices*, vol. lxiii. p. 93.

† *Astr. Nachr.*, Nos. 3163, 3451, 3591; and elsewhere.

Professor Harzer,* who has recently added some developments of the theory. In this problem the points which possess a zone of condensation are the poles of the great circles along which the observed proper motions take place. If then the plane of condensation can be determined by the method thus described, the apex of the solar motion is found as the pole of this plane. When different weights are assigned to the poles of the proper motions, these are to be used in finding the momental ellipsoid.

This method, at first sight so simple and elegant, is not free from difficulty and has been criticised in particular by M. Radau.† The difficulty is due to the absence of distinction between those poles which correspond to direct proper motions (with components directed towards the antiapex) and those which correspond to retrograde proper motions (with components directed towards the apex). If the position of the apex were known the two classes of poles could be discussed separately, and from the two results a weighted combination could be deduced which would represent the resultant of evidence in itself contradictory. Since the position of the apex, so far from being known, is precisely what has to be determined, the distinction between the two classes cannot be drawn directly, and we are reduced to the necessity of making a series of successive approximations. This method, though laborious, seems perfectly feasible.

In using the general method Dr. Kobold ignored the distinction between direct and retrograde proper motions. This procedure is equivalent to reversing the proper motions of those stars whose motion is retrograde. His result as regards the declination of the apex is in marked disagreement with the results found by other astronomers who used other methods, for it places the apex practically on the equator. By taking averages for stars in the same region of the sky instead of treating the proper motions separately, he obtained a result in better accord with other determinations. Hence it has been concluded that the discrepancy is to be attributed to the faulty nature of the method employed by Dr. Kobold. The whole question of the method which ought to be used for the determination of the apex of the solar motion has been the subject of much controversy, into which it is not intended to enter. But the discrepancy presented by Dr. Kobold's results cannot be passed over without some further consideration.

The axiom which lies at the base of all determinations of the apex is that the intrinsic proper motions of the stars exhibit no particular directed tendency, but possess in the main the character of errors of observation. If this be true, a simple consequence follows which is easily expressed in terms of the dynamical analogy employed in § 1. For it is evident that if we find the

* *Astr. Nachr.*, Nos. 3173 and 3998; also W. T. Carrigan, *Astr. Journ.*, No. 565.

† *Bull. Astr.*, vol. x. p. 401.

momental ellipsoids for (1) the direct proper motions, (2) the retrograde proper motions, (3) all proper motions irrespective of sense, then (1) and (2) ought to be ellipsoids of revolution of which the axes of symmetry should be directed to the apex of the solar motion. If this be the case the axis of (3), which corresponds to Dr. Kobold's procedure, will be at least a near approximation to the true solution of the problem. If it be not the case the ellipsoids (1) and (2) are either not surfaces of revolution or their axes are not coincident, the latter being the form to which M. Radau reduces his criticism of the method. Now either of these suppositions directly refutes the fundamental axiom. Hence the conclusion seems to be that if the material on which the discussion is based were perfectly satisfactory, even the method adopted by Dr. Kobold would be a proper one. That it leads to a conspicuous discrepancy with the results of other methods suggests strongly that the difficulty should be located not so much in the merits or demerits of a particular method as in the nature of the proper motions employed. And Professor Harzer * deduces from some unpublished figures communicated by Dr. Kobold that the errors are not all accidental.

Assuming that there is strong evidence that the intrinsic proper motions deviate in a marked degree from a purely accidental character, we have to inquire whether the deviations are real or whether they are only apparent and due to systematic errors in the observations and reductions. The latter possibility presents a subject too extensive and too remote from our purpose to discuss here. But the former may be considered briefly, because it seems not to have been clearly stated in the literature of the subject.

If we knew the annual parallaxes, proper motions, and motions in the line of sight of a number of stars, we could determine accurately the motion of the Sun relative to the mean centre of those stars. In the absence of such knowledge we have to attempt to do the same thing with inferior means. This is the basis of the methods of Bravais and Airy. Hence it is necessary to make some hypothesis such as the axiom already mentioned, which is that postulated by Professor Kapteyn † in his researches. It is clear that this would be appropriate if we could use the analogy of the kinetic theory of gases and could compare the relative motion of the Sun to the motion of a particular molecule relative to a single homogeneous stream of gas. But this supposition is apparently contradicted by the result of Dr. Kobold's application of Bessel's method. Hence we seem led to entertain at least the possibility that we have to deal not with one stream but with two or more distinct streams which it will be a matter of extreme difficulty to disentangle. It is known that proper motions are not distributed at random over the sky, and it has been usual in determinations of the apex to omit groups of

* *Astr. Nachr.*, No. 3998, p. 236. † *Astr. Nachr.*, Nos. 3721–2.

common proper motions as in the *Pleiades*. But such groups may be only particular cases of a feature characterising all proper motions and destructive of any uniform hypothesis.

If this view is correct, the physical significance of what is known as the motion of the Sun in space is greatly modified, for a velocity relative to two or more streams has no meaning. A great extension of optical theory and experiment may reveal the motion of the Sun relative to the ether, but this would have no bearing on the present question. The direction of the motion as determined by statistical methods can have merely a statistical meaning, and the agreement between the results of different methods is due to the fact that the methods are statistically equivalent. And since determinations of the constant of precession are affected by the corrections adopted for the parallactic motions, this constant also cannot be regarded as that which strictly corresponds to the dynamical conditions from which as a physical quantity it results.

3. A third problem which can be solved by the use of the same general method is that of the determination of meteor radiants. The radiant of the *Leonids* in 1899 was found in this way by Dr. Kobold.* In this application there is, of course, no such difficulty as occurs in the determination of the solar apex.

The object of this note has been to point out the interpretation of a method originally due to Bessel in terms of a momental ellipsoid, and the application of this single principle to three distinct problems : (1) the distribution of stars possessing a zone of concentration, (2) the determination of the apex of the Sun's motion, and (3) the determination of meteor radiants. The discordant result found on applying the method to the second problem necessitated a digression in which the cause of the discrepancy has been briefly discussed. It is concluded that there is a pronounced departure from an accidental distribution in the intrinsic proper motions of the stars, and it is suggested that we have to deal with two or more independent streams within which the relative motions may be distributed more or less in accordance with the normal law of errors.

University Observatory, Oxford :
1905 *April* 7.

* *Astr. Nachr.*, No. 3608, p. 120.

Discussion of the Observations of the Satellite of Neptune made at the Royal Observatory, Greenwich, in the years 1902–3–4. By F. W. Dyson, M.A., F.R.S., and D. J. R. Edney.

The observations discussed in this paper are published in the *Monthly Notices*, vols. lxii., lxiii., and lxiv., and are obtained from photographs with the 26-inch refractor taken with the aid of an occulting shutter. Specimen photographs are given in *Monthly Notices*, vol. lxii. p. 623, with an account of the occulting shutter and method and details of measurement. The published measures are compared with tabular places obtained from the *Connaissance des Temps* based on Mr. Hermann Struve's elements.

The notation and formulæ employed in obtaining corrections to the elements are taken from Mr. Struve's discussion in the *Mémoires de l'Académie Impériale des Sciences de St.-Pétersbourg*, vii^e^ série, tome xlii., No. 4. As Mr. Struve has discussed the observations prior to 1892, and as his elements are employed in the *Connaissance des Temps*, it is desirable and convenient to keep closely to his form. The only difference in notation is that, following the *Connaissance*, $+U$ is written where Mr. Struve has $-U$.

Assuming the orbit of the satellite to be circular, its distance s and position-angle p are given by the formulæ

$$s \sin (p-P) = r \sin (u+U)$$

$$s \cos (p-P) = r \sin B \cos (u+U)$$

$$r = a \frac{(\rho)}{\rho'}\ ;\ \rho' = \rho(1 + a \cos B \cos (u+U) \sin 1'')$$

where a is the elongation at mean distance (ρ) of *Neptune* ;

ρ is the geocentric distance of *Neptune* ;

u is the longitude of the satellite measured from the node of its orbit on the Earth's equator ;

$180° - U$ and B are the planetocentric longitude and latitude of the Earth with reference to the orbit of the satellite ;

and P is the position-angle of the pole of the orbit of the satellite ;

U, B, P are readily expressible in terms of N, I, the longitude of the node and inclination, and α, δ the right ascension and declination of the planet. Their values along with $a \frac{(\rho)}{\rho}$ and u are tabulated in the *Connaissance des Temps*.

The formulæ for correcting the elements are given by Mr. H. Struve in the form

$$\begin{aligned} s \sin dp = {} & r \sin \tau \,.\, \sin du \\ & + (r \sin \tau \cos \mathrm{I} + r \cos \tau \cos u \sin \mathrm{I}) \,.\, \sin d\mathrm{N} \\ & - r \cos \tau \sin u \,.\, \sin d\mathrm{I} \\ & - r \sin \tau \cos u \,.\, 2e \sin \mathrm{Q} \\ & + r \sin \tau \sin u \,.\, 2e \cos \mathrm{Q} \,; \\ ds = {} & r \cos \sigma \cos \tau \,.\, \sin du \\ & + r \cos \sigma \sin p \cos \delta \,.\, \sin d\mathrm{N} \\ & + r \cos \sigma \sin \tau \sin u \,.\, \sin d\mathrm{I} \\ & - \left(r \cos \sigma \cos \tau \cos u + \frac{s}{2} \sin u \right) . \; 2e \sin \mathrm{Q} \\ & + \left(r \cos \sigma \cos \tau \sin u - \frac{s}{2} \cos u \right) . \; 2e \cos \mathrm{Q} \\ & + s \,.\, \frac{da}{a} \end{aligned}$$

where e is the eccentricity, and Q the longitude of periastron measured from the node of the satellite's orbit on the Earth's equator.

And the auxiliaries τ and σ are defined by the equations :

$$\sin \tau = \frac{r}{s} \sin \mathrm{B}$$

$$\cos \tau = \frac{r}{s} \cos \mathrm{B} \sin (u + \mathrm{U})$$

$$\cos \sigma = \cos \mathrm{B} \cos (u + \mathrm{U}).$$

The adopted value of N and I in the above formulæ are

$$\mathrm{N} = 185^{\circ}\cdot 15 + 0^{\circ}\cdot 148 \; (t - 1890)$$

$$\mathrm{I} = 119\cdot 35 - 0\cdot 165 \; (t - 1890)$$

The equations of condition obtained by applying the above formulæ to the results of the Greenwich photographs are given below for the three oppositions considered. The residuals are given in the last column. Taking $e=0$ and adopting the mean values for du, $d\mathrm{N}$, $d\mathrm{I}$, and $\frac{da}{a}$, the probable error for weight 1, *i.e.* for a result derived from one photograph, is $\pm 0''\cdot 14$ for both the distances and position-angles.

Equations of Condition.

Position-angle.

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	$2e \sin Q$.	$2e \cos Q$.	$s \sin dp$.	Residual.
1902.								
Jan. 6	2	−14″·1	+1″·0	−6″·9	− 9″·5	−10″·5 =	+0″·18	−0″·05
14	2	−15·9	+6·7	+5·6	− 2·2	+15·8 =	+ ·21	+ ·01
24	2	−12·5	−2·8	−5·9	−10·7	− 6·5 =	+ ·35	+ ·11
28	2	−11·8	−2·7	+7·9	+ 8·9	− 7·7 =	+ ·29	+ ·10
29	2	−16·8	+7·8	+0·3	− 2·6	−16·6 =	+ ·36	+ ·14
30	1	−11·9	−4·2	−4·9	−10·8	− 4·9 =	+ ·39	+ ·15
31	1	−11·5	−3·7	+6·9	− 9·5	+ 6·4 =	·00	− ·21
Feb. 10	2	−16·4	+6·6	−3·0	− 5·4	−15·5 =	+ ·21	− ·02
11	4	−11·2	−5·6	−2·7	−10·9	− 2·5 =	+ ·20	− ·03
12	1	−12·2	−1·1	+8·6	− 8·1	+ 9·1 =	+ ·14	− ·05
13	1	−16·3	+6·3	−3·4	+ 5·7	+15·2 =	+ ·36	+ ·13
15	2	−12·6	+0·1	+8·8	+ 7·4	−10·2 =	+ ·13	− ·06
16	2	−16·1	+5·9	−4·1	− 6·3	−14·8 =	+ ·44	+ ·20
28	3	−14·0	+1·5	−6·6	− 9·2	−10·7 =	·00	− ·23
Mar. 1	1	−10·6	−6·2	+2·5	−10·4	+ 2·1 =	+ ·53	+ ·32
3	5	−13·7	+0·7	−6·6	+ 9·5	+ 9·9 =	+ ·17	− ·06
17	1	−16·3	+7·6	+1·8	− 1·4	−16·3 =	+ ·41	+ ·20
19	3	−11·0	−4·0	+6·7	− 9·2	+ 6·0 =	+ ·13	− ·07
21	3	−11·5	−4·2	−4·6	+10·6	+ 4·5 =	+ ·24	+ ·01
22	1	−11·4	−2·8	+7·6	+ 8·7	− 7·3 =	+ ·15	− ·04
25	3	−11·4	−2·7	+7·7	− 8·6	+ 7·4 =	+ ·27	+ ·08
27	4	−11·1	−5·2	−3·3	+10·6	+ 3·0 =	+ ·20	− ·03
Apr. 6	1	−12·4	+0·4	+8·6	− 7·1	+10·2 =	+ ·08	− ·10
10	2	−14·8	+3·8	−5·6	− 7·7	−12·7 =	+ ·22	·00

Equations of Condition.

Distance.

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	$2e \sin Q$.	$2e \cos Q$.	$\frac{da}{a}$.	$s \sin dp$.	Residual.
1902.									
Jan. 6	2	+6″·0	−8″·1	− 6″·8	−0″·7	+8″·7	+12″·9 =	+0″·10	+″·06
14	2	−4·1	+3·2	−11·6	+5·0	+4·9	+11·4 =	+ ·23	+·10
24	2	+5·7	−7·3	− 3·3	+1·1	+9·1	+14·5 =	+ ·09	+·04
28	1	−4·8	+5·4	− 3·1	−1·2	−8·8	+15·4 =	+ ·12	−·01
29	2	−0·2	−1·7	−12·7	−5·3	+0·6	+10·7 =	+ ·08	−·02
30	1	+5·0	−6·3	− 2·1	+1·5	+8·9	+15·2 =	− ·06	−·11
31	1	−4·2	+4·8	− 2·2	+1·0	+9·0	+15·9 =	− ·21	−·32
Feb. 10	2	+2·4	−4·7	−11·8	−4·4	+4·1	+10·9 =	− ·08	−·15

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	$2e \sin Q$.	$2e \cos Q$.	$\frac{da}{a}$.	$s \sin dp$.	Residual.
1902.									
Feb. 11	4	+3″·7	−4″·5	− 0″·7	+1″·8	+8″·6	+16″·0 =	+ ″·03	−″·02
12	1	−5·5	+5·9	− 4·4	+1·8	+9·0	+14·6 =	− ·11	−·25
13	1	+2·8	−5·1	−11·5	+4·2	−4·5	+10·9 =	+ ·28	+·21
15	2	−5·8	+6·2	− 5·5	−2·3	−8·9	+14·1 =	+ ·22	+·09
16	2	+3·3	−5·7	−11·1	−3·8	+5·2	+11·1 =	+ ·07	·00
28	3	+5·7	−8·1	− 7·1	−1·0	+8·4	+12·4 =	+ ·17	+·12
Mar. 1	1	−0·2	+0·2	0·0	+1·4	+8·3	+16·6 =	+ ·15	+·06
3	5	+5·9	−8·1	− 6·3	+0·5	−8·6	+12·7 =	+ ·03	−·01
17	1	−1·4	−0·2	−12·5	− 5·4	−0·9	+10·5 =	− ·17	−·28
19	3	−3·7	+4·2	− 1·8	+1·2	+8·5	+15·6 =	+ ·21	+·10
21	3	+4·9	−6·2	− 1·9	−1·6	−8·8	+14·8 =	+ ·10	+·05
22	1	−4·6	+5·1	− 2·8	−1·4	−8·7	+15·1 =	+ ·23	+·10
25	3	−4·6	+5·1	− 2·9	+1·4	+8·7	+15·0 =	+ ·06	−·07
27	4	+4·0	−5·0	− 1·0	−1·8	−8·5	+15·4 =	− ·06	−·11
Apr. 6	1	−5·7	+6·1	− 5·6	+2·3	+8·6	+13·6 =	+ ·24	+·12
10	2	+4·7	−7·1	− 9·1	−2·4	+6·9	+11·3 =	+ ·22	+·17

Equations of Condition.

Position-angle.

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	$2e \sin Q$.	$2e \cos Q$.	$s \sin dp$.	Residual.
1902.								
Nov. 12	4	−12″·5	−1″·3	+7″·9	+ 8″·9	− 8″·8 =	−0″·15	−″·27
13	2	−16·5	+6·9	−3·0	− 3·8	−16·1 =	− ·19	−·35
17	1	−11·2	−5·8	−2·7	+10·9	+ 2·4 =	+ ·49	+·31
28	2	−14·7	+2·6	−6·9	+ 8·1	+12·5 =	− ·04	−·23
Dec. 29	2	−11·6	−4·1	+6·4	+ 9·9	− 6·0 =	− ·13	−·26
31	2	−11·6	−5·1	−4·1	−10·9	− 3·9 =	+ ·06	−·13
1903.								
Jan. 1	1	−12·7	−0·9	+8·3	− 8·6	+ 9·4 =	− ·03	−·14
3	2	−11·5	−5·2	−3·9	+10·9	+ 3·7 =	+ ·64	+·45
15	3	−11·0	−6·4	−0·3	+11·0	+ 0·3 =	+ ·28	+·11
23	3	−14·5	+2·0	−6·9	− 8·7	−11·6 =	+ ·50	+·30
25	1	−14·6	+3·9	+7·7	− 5·5	+13·6 =	− ·28	−·38
28	2	−14·7	+4·1	+7·6	+ 5·3	−13·7 =	+ ·29	+·19
Feb. 1	1	−13·2	−0·8	−6·9	+ 9·9	+ 8·8 =	+ ·77	+·58
2	1	−10·9	−6·0	+3·4	+10·5	− 2·9 =	+ ·22	+·07
6	1	−16·1	+6·9	+4·6	− 2·0	+16·0 =	+ ·61	+·49
10	1	−12·3	−3·1	−5·9	−10·5	− 6·4 =	+ ·07	−·12
16	2	−11·4	−5·2	−3·7	−10·8	− 3·5 =	+ ·19	·00
17	3	−11·5	−3·6	+6·8	− 9·5	+ 6·5 =	− ·05	−·17

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	$2e$ sin Q.	$2e$ cos Q.		s sin *dp*.	Residual.
1902.									
Feb. 18	3	−16″·7	+7″·4	−1″·3	+ 3″·2	+16″·4	=	+ ″·15	″·00
23	1	−11·8	−2·6	+7·6	− 9·0	+ 7·6	=	+ ·27	+·15
26	1	−12·2	−1·4	+8·1	+ 8·5	− 8·8	=	+ ·21	+·10
28	3	−10·9	−5·9	−2·0	−10·8	− 1·7	=	+ ·29	+·11
Mar. 3	3	−10·9	−6·0	−1·6	+10·8	+ 1·4	=	− ·03	−·21
6	3	−10·7	−6·3	−0·3	−10·7	− 0·3	=	+ ·22	+·06
11	2	−14·6	+2·9	−6·4	− 8·1	−12·1	=	·00	−·18
13	1	−13·9	+3·1	+8·0	− 5·9	+12·7	=	+ ·12	+·02
15	2	−10·6	−6·0	+2·0	+10·5	− 1·6	=	+ ·23	+·08
16	2	−14·9	+5·2	+6·7	+ 4·0	−14·4	=	·00	−·10
21	2	−10·7	−5·8	+3·5	+10·2	− 3·0	=	− ·11	−·25
26	1	−12·4	−2·0	−6·3	+10·0	+ 7·3	=	+ ·36	+·18
Apr. 14	1	−11·8	−3·7	+7·7	+ 8·4	− 8·2	=	+ ·47	+·34
16	1	−10·8	−5·5	−2·6	−10·5	− 2·4	=	+ ·51	+·33
17	1	−12·1	−0·7	+8·0	− 8·0	+ 9·1	=	− ·04	−·14
24	1	−14·8	+4·0	−5·7	+ 7·0	+13·0	=	+ ·18	+·01
27	1	−14·3	+3·1	−6·2	− 7·6	−12·1	=	+ ·22	+·04

Equations of Condition.

Distance.

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	$2e$ sin Q.	$2e$ cos Q.	$\frac{da}{a}$.		s sin *dp*.	Residual.
1902.										
Nov. 12	4	−5″·6	+6″·5	− 4″·4	−1″·2	−9″·1	+14″·6	=	+0″·10	−″·02
13	2	+2·3	−3·6	−12·1	−4·9	+3·5	+11·1	=	+ ·18	+·07
17	1	+2·8	−3·4	− 0·5	−0·9	−8·6	+16·4	=	− ·31	−·39
28	2	+5·5	−7·3	− 8·3	+2·2	−8·1	+12·6	=	− ·15	−·25
Dec. 29	2	−4·1	+4·8	− 2·0	−0·7	−8·9	+16·0	=	+ ·28	+·16
31	2	+4·0	−5·0	− 1·3	+1·1	+8·9	+16·0	=	+ ·19	+·11
1903.										
Jan. 1	1	−5·7	+6·5	− 4·8	+1·5	+9·1	+14·6	=	+ ·02	−·11
3	2	+3·9	−4·9	− 1·2	−1·2	−8·9	+16·1	=	+ ·19	+·11
15	3	+1·3	−1·6	0·0	−1·1	−8·4	+16·8	=	·00	−·08
23	3	+5·7	−7·7	− 7·7	−1·7	+8·4	+12·7	=	·00	−·09
25	1	−5·6	+5·8	− 9·1	+3·7	+7·5	+12·5	=	− ·19	−·33
28	2	−5·5	+5·7	− 9·3	−3·8	−7·4	+12·4	=	+ ·38	+·24
Feb. 1	1	+5·9	−7·7	− 5·0	+0·2	−9·1	+13·8	=	− ·13	−·21
2	1	−1·5	+1·7	− 0·3	−0·8	−8·4	+16·7	=	+ ·01	−·09
6	1	−3·4	+2·8	−11·9	+5·2	+4·1	+11·3	=	− ·11	−·25
10	1	+5·4	−6·9	− 3·0	+0·8	+9·1	+14·7	=	− ·49	−·58
16	2	+3·9	−4·9	− 1·1	+1·3	+8·7	+15·8	=	+ ·23	+·15

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	e sin Q.	e cos Q.	$\frac{da}{a}$.		s sin *dp*.	Residual.
1902.										
Feb. 17	3	−4·2	+4·9	− 2·3	+0·9	+8·8	+15·6	=	+ ·20	+·08
18	3	+1·0	−2·6	−12·5	+5·1	−2·0	+10·8	=	− ·01	−·14
23	1	−4·9	+5·6	− 3·2	+1·2	+8·9	+15·1	=	− ·02	−·14
26	1	−5·4	+6·1	− 4·2	−1·5	−9·0	+14·6	=	+ ·16	+·04
28	3	+2·7	−3·3	− 0·4	+1·4	+8·4	+16·2	=	+ ·30	+·22
Mar. 3	3	+2·4	−3·0	− 0·3	−1·4	−8·4	+16·2	=	+ ·10	+·02
6	3	+1·5	−1·8	0·0	+1·3	+8·3	+16·4	=	+ ·19	+·11
11	2	+5·3	−7·4	− 8·3	−2·1	+7·7	+12·0	=	+ ·27	+·17
13	1	−5·7	+6·0	− 8·2	+3·3	+7·8	+12·6	=	+ ·21	+·07
15	2	−0·3	+0·3	0·0	−1·0	−8·2	+16·4	=	+ ·18	+·09
16	2	−4·9	+4·7	−10·1	−4·3	−6·3	+11·7	=	− ·15	−·30
21	2	−1·5	+1·8	− 0·4	−0·9	−8·2	+16·3	=	+ ·56	+·46
26	1	+5·6	−7·3	− 3·9	−0·4	−8·9	+13·9	=	+ ·16	+·07
Apr. 14	1	−5·1	+5·8	− 3·8	−1·3	−8·7	+14·3	=	− ·04	−·16
16	1	+3·1	−3·8	− 0·6	+1·3	+8·3	+15·6	=	− ·02	−·10
17	1	−5·4	+6·1	− 4·7	+1·7	+8·7	+13·9	=	− ·15	−·27
24	1	+4·6	−6·5	− 9·2	+2·8	−6·7	+11·3	=	− ·01	−·11
27	1	+5·0	−7·0	− 8·4	−2·3	+7·4	+11·6	=	− ·17	−·26

Equations of Condition.

Position-angle.

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	e sin Q.	e cos Q.		s sin *dp*.	Residual.
1903.									
Dec. 4	1	−11·2	−5·9	−2·8	+10·9	+ 2·4	=	+0·13	−·11
8	1	−13·9	+1·4	+7·9	− 7·8	+11·4	=	+ ·26	+·11
9	2	−15·4	+4·4	−6·3	+ 6·5	+14·0	=	+ ·25	+·07
10	2	−11·1	−6·4	−1·1	+11·0	+ 0·9	=	+ ·33	+·09
14	1	−14·2	+2·1	+7·9	− 7·4	+12·1	=	+ ·11	−·04
15	1	−14·6	+2·6	−7·2	+ 7·8	+12·4	=	+ ·29	+·08
17	1	−14·5	+2·9	+7·7	+ 6·9	−12·7	=	+ ·05	−·09
30	2	−12·8	−1·8	−7·0	− 9·9	− 8·1	=	+ ·22	−·01
31	2	−11·4	−5·2	+4·6	−10·6	+ 4·2	=	+ ·29	+·07
1904.									
Jan. 6	1	−11·6	−4·4	+5·7	−10·3	+ 5·4	=	+ ·26	+·05
13	2	−16·9	+7·7	+0·3	+ 0·7	+16·9	=	+ ·23	+·08
14	2	−11·7	−4·6	−4·9	+10·8	+ 4·7	=	+ ·27	+·02
15	3	−12·2	−2·7	+7·2	+ 9·6	− 7·5	=	+ ·26	+·08
19	2	−16·6	+7·0	−2·9	+ 3·6	+16·2	=	+ ·15	−·01
22	1	−16·7	+7·2	−2·3	− 3·1	−16·4	=	+ ·16	·00
Feb. 2	1	−14·0	+2·0	+8·0	+ 7·2	−12·0	=	− ·23	−·37

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	$2e \sin Q$.	$2e \cos Q$.	$s \sin dp$.	Residual.
1904.								
Feb. 5	4	−14″·2	+2″·7	+7″·9	− 6″·8	+12″·5 =	+ ″·13	−″·02
6	1	−14·6	+2·6	−6·9	+ 8·0	+12·2 =	+ ·07	−·13
13	2	−10·9	−6·3	+1·8	+10·8	− 1·6 =	+ ·18	−·05
15	2	−13·4	−0·2	−7·2	− 9·4	− 9·5 =	+ ·30	+·07
17	2	−15·8	+6·1	+5·3	− 3·5	+15·4 =	+ ·05	−·08
Mar. 10	2	−11·3	−5·1	−3·9	−10·6	− 3·7 =	+ ·13	−·10
11	4	−12·3	−1·5	+7·7	− 8·8	+ 8·6 =	+ ·19	+·02
21	2	−14·8	+3·7	−6·2	− 7·1	−13·0 =	+ ·31	+·12
Apr. 6	2	−10·7	−5·7	+3·2	+10·3	− 2·8 =	+ ·26	+·04
12	1	−10·8	−5·0	+4·6	+10·0	− 4·1 =	+ ·34	+·14
16	2	−16·0	+7·2	+1·8	− 0·4	+16·0 =	+ ·16	+·03
18	2	−11·2	−3·8	+6·1	+ 9·5	− 5·8 =	−0·02	−·21

Equations of Condition.

Distance.

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	$2e \sin Q$.	$2e \cos Q$.	$\frac{da}{a}$.	$s \sin dp$.	Residual.
1903.									
Dec. 4	1	+2″·4	−3″·0	− 0″·5	−0″·5	−8″·6	+16″·6 =	+0″·10	+″·07
8	1	−5·9	+6·9	− 7·0	+2·2	+8·6	+13·4 =	+ ·12	+·07
9	2	+4·8	−6·3	− 9·8	+3·4	−6·9	+12·1 =	− ·15	−·21
10	2	+1·2	−1·4	− 0·1	−0·5	−8·5	+16·8 =	− ·08	−·11
14	1	−5·8	+6·8	− 7·6	+2·6	+8·4	+13·1 =	+ ·10	+·05
15	1	+5·6	−7·1	− 8·2	+2·4	−8·1	+12·7 =	+ ·04	−·03
17	1	−5·7	+6·6	− 8·3	−2·9	−8·1	+12·8 =	− ·21	−·27
30	2	+5·7	−7·1	− 4·1	−0·3	+9·2	+14·6 =	+ ·10	+·05
31	2	−3·1	+3·7	− 1·0	+0·2	+8·8	+16·4 =	− ·09	−·12
1904.									
Jan. 6	1	−3·9	+4·7	− 1·7	+0·3	+8·9	+16·0 =	+ ·21	+·17
13	2	−0·3	−0·4	−12·8	+5·5	0·0	+11·0 =	− ·13	−·20
14	2	+4·2	−5·2	− 1·6	−0·7	−8·9	+15·8 =	− ·19	−·24
15	3	−5·0	+6·0	− 3·2	−0·7	−9·1	+15·3 =	+ ·02	−·02
19	2	+2·3	−3·4	−12·2	+5·0	−3·4	+11·2 =	+ ·18	+·11
22	1	+1·8	−2·9	−12·4	−5·1	+2·8	+11·1 =	− ·18	−·25
Feb. 2	1	−5·8	+6·6	− 7·4	−2·6	−8·4	+13·2 =	+ ·09	+·03
5	4	−5·7	+6·4	− 8·0	+3·0	+8·1	+12·9 =	+ ·07	+·01
6	1	+5·5	−7·2	− 8·1	+2·2	−8·0	+12·5 =	+ ·37	+·30
13	2	−0·7	+0·8	− 0·1	−0·6	−8·4	+16·7 =	+ ·04	+·01
15	2	+5·8	−7·5	− 5·6	−0·8	+8·9	+13·6 =	+ ·08	+·02
17	2	−4·0	+4·1	−11·2	+4·7	+5·1	+11·5 =	+ ·06	−·02

Date.	Weight.	sin *du*.	sin *dN*.	sin *dI*.	$2e \sin Q$.	$2e \cos Q$.	$\frac{da}{a}$.	$s \sin dp$.	Residual.
1903.									
Mar. 10	2	+3″·7	−4″·6	− 1″·1	+0″·9	+8″·6	+15″·7 =	+ ″·06	+″·02
11	4	−5·4	+6·2	− 4·2	+1·2	+8·9	+14·4 =	− ·09	−·13
21	2	+4·9	−6·6	− 9·0	−2·8	+7·1	+11·8 =	+ ·24	+·18
Apr. 6	2	−1·7	+2·1	− 0·4	−0·4	−8·2	+16·1 =	+ ·36	+·34
12	1	−2·8	+3·4	− 1·0	−0·4	−8·4	+15·7 =	+ ·26	+·23
16	2	−1·4	+0·9	−12·1	+5·3	+1·5	+10·6 =	+ ·09	+·02
18	2	−4·0	+4·8	− 2·0	−0·5	−8·6	+15·1 =	+ ·22	+·18

Normal Equations.

1902.

	sin *du*.	sin *dN*.	sin *dI*.	$2e \sin Q$.	$2e \cos Q$.	$\frac{da}{a}$.		
sin *du*	9927	−1504	+ 96	+1131	+1189	+ 930	=	−148·14
sin *dN*		2684	+ 687	+ 139	− 660	−1602	=	− 4·44
sin *dI*			3935	+ 141	+ 120	−3295	=	− 31·88
$2e \sin Q$				4241	+1516	− 169	=	− 19·34
$2e \cos Q$					8059	+1260	=	− 15·47
$\frac{da}{a}$						9526	=	+ 53·08

1902–03.

	sin *du*.	sin *dN*.	sin *dI*.	$2e \sin Q$.	$2e \cos Q$.	$\frac{da}{a}$.		
sin *du*	11430	− 673	− 400	− 406	+1106	+ 222	=	−123·09
sin *dN*		3060	+ 158	− 280	− 301	− 618	=	− 18·17
sin *dI*			4120	+ 528	− 69	− 3471	=	− 36·90
$2e \sin Q$				5479	+ 628	− 86	=	+ 7·39
$2e \cos Q$					9307	− 590	=	+ 1·77
$\frac{da}{a}$						13333	=	+ 93·48

1903–4.

	sin *du*.	sin *dN*.	sin *dI*.	$2e \sin Q$.	$2e \cos Q$.	$\frac{da}{a}$.		
sin *du*	10146	−1547	− 710	− 108	−2758	− 581	=	−127·67
sin *dN*		2468	− 175	− 334	+1233	+ 496	=	− 5·87
sin *dI*			4232	− 715	+ 230	− 3579	=	− 8·18
$2e \sin Q$				3956	− 210	+ 563	=	+ 2·70
$2e \cos Q$					8714	− 12	=	+ 26·23
$\frac{da}{a}$						10074	=	+ 34·39

Solutions.

	1902.	1902–3.	1903–4.	Mean.
sin *du*	−0·01653	−0·01160	−0·01458	−0·0142
sin *d*N	−0·00736	−0·00688	−0·01250	−0·0089
sin *d*I	−0·00196	−0·00512	−0·00349	−0·0035
2*e* sin Q	+0·00051	+0·00053	−0·00167	−0·0002
2*e* cos Q	−0·00039	+0·00163	+0·00022	+0·0005
$\frac{da}{a}$	+0·00533	+0·00563	+0·00201	+0·0043
du	−0°·95	−0°·67	−0°·83	−0°·82
*d*N	−0·42	−0·40	−0·72	−0·50
*d*I	−0·12	−0·30	−0·20	−0·20
da	+0″·087	+0″·092	+0″·033	+0″·070

The Eccentricity of the Orbit.—The three determinations show that the eccentricity is extremely small, the value actually found for 2*e* sin Q and 2*e* cos Q being smaller than their probable errors. Mr. H. Struve gave ·01 as the maximum possible value of the eccentricity. The present observations show the limit may be put very much lower. The following values for 2*e* sin Q and 2*e* cos Q derived from Dr. Struve's *Memoir* are given for comparison:

Observer.	Epoch.	2*e* sin Q.	2*e* cos Q.
Newcomb	1874·5	−·0006	−·0176
Hall	1876·3	−·0068	−·0167
Holden	1876·5	+·0102	+·0002
Hall	1882·1	+·0056	−·0038
Hall	1883·8	+·0100	−·0173
A. Hall (junior)... ...	1892·0	−·0064	−·0168
H. Struve I.	1887·6	−·0100	−·0005
„ II.	1889·0	−·0139	−·0056
„ III.	1890·6	−·0144	+·0002
„ IV.	1892·6	−·0154	−·0062
Greenwich	1902·1	+·0005	−·0004
„	1903·1	+·0005	+·0016
„	1904·1	−·0017	+·0002

The figures suggest that there are in the visual observations some systematic effects due to personal equation from which the photographic observations are free, and that the eccentricity of the orbit is less than 0·001.

Rejecting the terms $2e \sin Q$ and $2e \cos Q$ and combining the three series of normal equations we obtain

	$\sin du$.	$\sin dN$.	$\sin dI$.	$\frac{da}{a}$.	
$\sin du$	31503	−3724	− 1014	+ 571	= −398·90
$\sin dN$		+8212	+ 670	− 1724	= − 28·48
$\sin dI$			+12287	−10345	= − 76·96
$\frac{da}{a}$				+32933	= +180·95

These equations give for the epoch 1903·1 :

$$\sin du = -·0139 \pm ·00081 \quad du = -0^{\circ}·80 \pm 0^{\circ}·08$$
$$\sin dN = -·0086 \pm ·00193 \quad dN = -0^{\circ}·50 \pm 0^{\circ}·11 \quad N = 187^{\circ}·58$$
$$\sin dI = -·0034 \pm ·00128 \quad dI = -0^{\circ}·20 \pm 0^{\circ}·07 \quad I = 117^{\circ}·40$$
$$\frac{da}{a} = +·0042 \pm ·00093 \quad da = +0''·069 \pm 0''·015 \quad a = 16''·202$$

The Longitude and Mean Motion.—Comparison of the value $du = -0^{\circ}·80$ with the values given for different epochs in Dr. Struve's *Memoir* (p. 61) gives a small correction to the longitude for 1890 and to the mean daily motion. The adopted values for 1890·0 $u = 234^{\circ}·42$ and mean daily motion $n = 61^{\circ}·25748$ appear to require corrections of approximately $-0^{\circ}·20$ and $+0^{\circ}·00008$ respectively.

The Mean Distance of the Satellite and Mass of Neptune.—Owing to the great difference in magnitude of *Neptune* and its satellite it is possible that the visual measures are all affected by a small personal equation. The photographic determination is free from this difficulty; and as great care was taken in the determination of the value of the scale, the photographic results would seem specially valuable for this element.

The value found, $a = 16''·202$, corresponds to the mean distance of the planet for which $\log \rho = 1·47814$. The corresponding value of the mass of *Neptune* is $\frac{1}{M} = 19474$.

The following values were obtained by different observers with the Washington refractor and by Dr. Struve at Pulkowa :

	a	$\frac{1}{M}$
Newcomb	16·275	= 19382
Holden	16·598	18273
Hall 1875–77	16·482	18662
Hall 1881–82	16·368	19054
Hall 1883–84	16·263	19425
A. Hall (junior)	16·602	18260
H. Struve	16·271	19396
Greenwich...	16·202	19474

The Movement of the Plane of the Satellite's Orbit.—The values $dN = -0°{\cdot}50$, $dI = -0°{\cdot}20$, give for the epoch 1903·1 $N = 187°{\cdot}58$, $I = 117°{\cdot}40$. The following table obtained by taking the simple means of the results of different observations near the same epochs in a fuller table given by Dr. Struve shows the changes in the node and inclination since the discovery of the satellite :

Epoch.	N.	I.	Epoch.	N.	I.
1849·9	179°·12	126°·01	1883·0	184°·36	120°·08
1864·0	181·49	124·20	1890·4	185·27	119·16
1875·8	183·10	121·46	1903·1	187·58	117·40

These changes first pointed out by Marth were explained by Tisserand and Newcomb as arising from the spheroidal shape of *Neptune*. In consequence of the spheroidal figure the orbit will preserve a constant inclination to the equator of *Neptune*, and the node will revolve uniformly on *Neptune's* equator ; or in other words the pole of the orbit will describe uniformly a small circle round the pole of the planet. The interesting problem is thus presented of determining the direction of *Neptune's* axis, the inclination of the pole of the orbit to this axis, and the mechanical oblateness of *Neptune* from these changes in the node and inclination.

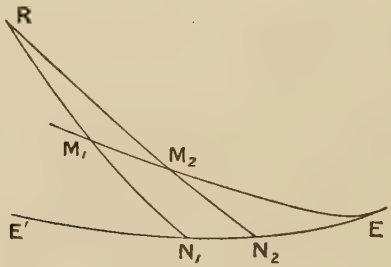


FIG. 1.

In the figure let E'E be the Earth's equator, RM_1N_1 the plane of the satellite's orbit for 1874·0, RM_2N_2 the plane of the orbit for 1896·5, and M_1M_2E the plane of *Neptune's* equator.

Let $\gamma = N_1M_1E$ = the inclination of the orbit of the satellite to *Neptune's* equator ;

θ_1 = longitude of this node on *Neptune's* equator at the epoch 1874·0 ; and

$\psi_1 = M_1N_1$ = the distance between the nodes of the orbit on the two planes of *Neptune's* equator and the Earth's equator.

The differential relations between θ, ψ, γ, and N, I, the node and inclination, are given by the equations

$$\left.\begin{aligned} \sin\gamma d\theta &= -\cos\psi \sin I dN + \sin\psi dI \\ d\gamma &= -\sin\psi \sin I dN - \cos\psi dI \end{aligned}\right\}$$

Putting $\dfrac{d\gamma}{dt} = 0$ and $\dfrac{d\theta}{dt} = \text{const.}$

we obtain $$\tan\psi = -\frac{dI}{\sin I dN}$$

In this way Dr. Struve finds for the epoch 1874·0, $\psi_1 = 52°{\cdot}6$ and $\sin\gamma d\theta = 0°{\cdot}208$, giving the period of revolution of the pole of the orbit round the pole of *Neptune* equal to 1734 sin γ years.

Comparing the present observations with Dr. Struve's in 1890, the changes of N and I in this interval and the value of ψ for the mean date 1896·5 is found :

(H. Struve)	1890·0	$N = 185°{\cdot}15$	$I = 119°{\cdot}35$
Greenwich	1903·1	$N = 187{\cdot}58$	$I = 117{\cdot}40$
	1896·5	$dN : dI = 2°{\cdot}43 : 1°{\cdot}95$	
		$N = 186°{\cdot}36$. $I = 118{\cdot}38$	

Therefore $\psi_2 = 41°{\cdot}7$ and $\sin\gamma d\theta = 0°{\cdot}224$ and the time of revolution = 1607 sin γ years.

Comparison of the values of ψ for 1874·0 and 1896·5 may be used to determine a rough value of γ, and of the position of *Neptune's* equator.

We have $N_1M_1 = 52°{\cdot}6$ $\quad N_2M_2 = 41°{\cdot}3 \pm 2°{\cdot}5$

$M_1N_1N_2 = 121{\cdot}91$ $\quad M_2N_2N_1 = 61{\cdot}62$

and $N_1N_2 = 3°{\cdot}40$

Solving the triangles we find

$$EM_2N_2 = \gamma = 22°$$

$$M_2EN_2 = 46°$$

and $$N_2E = 21°$$

Thus the inclination of the orbit to that of *Neptune* is 22°, and the longitude of the node and inclination of *Neptune's* to the Earth's equator are 207° and 134°.

This value of the inclination implies a rotation of the pole of the satellite's orbit in about 600 years. These results are extremely rough, depending as they do on changes in the small quantities dN and dI.

Graphic Solution of the Question.—The positions of the pole of the orbit are given by $\alpha = 90° + N$ and D = N.P.D. = 180° − I.

For the six epochs considered, the values of α and D are given in the second and third columns of the following table :

Epoch.	R.A. of Pole of Orbit $= \alpha$.	N.P.D. of Pole of Orbit D.	$\sin(\alpha-270°)\tan\frac{D}{2} = y$.	$\cos(\alpha-270°)\tan\frac{D}{2} = x$.
1849·9	269°·12	54°·00	−·008	+·510
1864·0	271·49	55·80	+·014	+·529
1875·8	273·10	58·53	+·030	+·559
1883·0	274·36	59·92	+·044	+·574
1890·4	275·27	60·83	+·054	+·585
1903·1	277·58	62·60	+·081	+·603

The best small circle is to be drawn through the six points. In order to exhibit this graphically let a stereographic projection be made by producing lines from the south pole through the six points to meet the tangent plane at the north pole. As a small circle on a sphere is projected into a small circle on the plane, the problem becomes one of plane geometry.

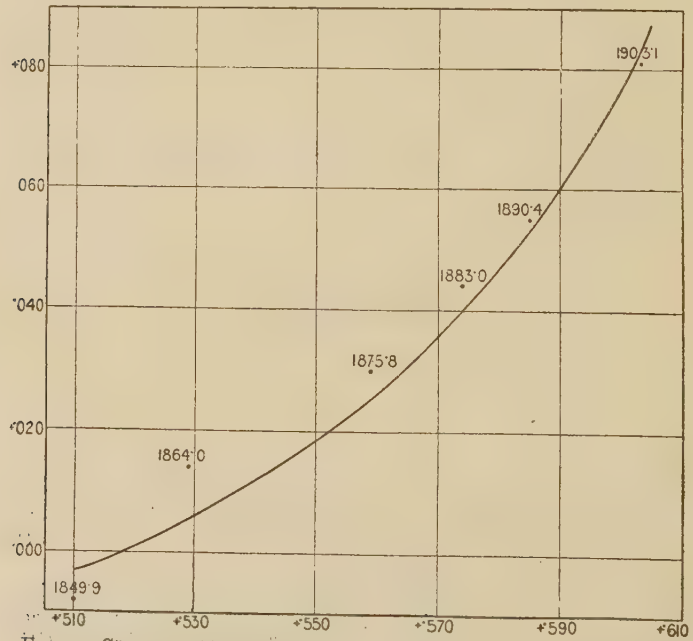


Fig. 2.—Stereographic Projection of Pole of Orbit of *Neptune's* Satellite.

$$x = \tan\frac{D}{2}\cos(\alpha-270°) \qquad y = \tan\frac{D}{2}\sin(\alpha-270°)$$

Taking the line whose right ascension is 270° as the initial line, the polar coordinates of the projection of the points a, D are $\tan \frac{D}{2}$ and $\alpha - 270°$, and the rectangular coordinates $\tan \frac{D}{2} \cos(\alpha - 270°)$ and $\tan \frac{D}{2} \sin(\alpha - 270°)$. These are tabulated in the fourth and fifth columns of the above table. Assuming the equation of the circle to be

$$x^2 + y^2 + fx + gy + c = 0$$

the values of f, g, and c are determined by a least-square solution, and the equation of the circle found to be

$$(x - \cdot 453)^2 + (y - \cdot 152)^2 = (\cdot 165)^2$$

This circle is the projection of the small circle whose centre is at R.A. 288° and N.P.D. 50°, and whose radius is 16°, giving 198° and 130° for the longitude of the node and equator of *Neptune*, and 16° for the inclination of the satellite's orbit to *Neptune's* equator.

The result obtained in this way agrees as closely as could be expected with that previously found, and probably closer accordance would have been obtained if more weight had been given to the later and more accurate observations. M. Tisserand, discussing the observations up to 1890, obtained entirely different results, and concluded that the inclination of the orbit of the satellite to the equator of *Neptune* was much greater. The present series of observations and those of Dr. Struve are of so much greater weight than the earlier observations that the value of the material for determining the inclination has been increased considerably. A glance at the diagram on p. 582 shows that observations have not yet been continued long enough for a determination of the inclination to be made with any degree of certainty. It is clearly of great importance that observations of the satellite should be continued, as it would seem that a series of good observations continued over another ten years would fix the position of *Neptune's* axis and the inclination of the orbit to it within a few degrees.

Reduction of the Right Ascensions of the Hong Kong Catalogue of 2120 Southern Stars for the Epoch 1900 to the System of Auwers' Southern Fundamental Stars. By A. M. W. Downing, D.Sc., F.R.S.

From the Preface to the Hong Kong Catalogue of right ascensions of southern stars, recently published, we learn that Auwers' Catalogue, contained in *Astron. Nachrichten*, Nos. 3431–2, was generally used as the standard, but that occasionally

the clock errors were determined "from some stars in the *Nautical Almanac*; but as soon as any of the stars in the Hong Kong Catalogue had been observed four times they were used as standard stars and given weight equal to Auwers' right ascensions." It becomes a matter of interest, therefore, to investigate the systematic differences between the right ascensions of the Hong Kong Catalogue and those of Auwers' Catalogue, and I now lay before the Society the results of the investigation.

There are 300 stars, situated between 20° and 60° of south declination, common to the catalogues, and the places of these have been compared, using the proper motions given by Auwers.

The mean difference in the sense Auwers *minus* Hong Kong is $+^{s}\cdot 0015$.

The first table gives the corrections to the Hong Kong right ascensions when the stars are arranged in order of declination, and taken in zones extending each to 5° of declination. These quantities have been graphically represented on cross-ruled paper, and an interpolating curve drawn in the usual manner, from which the correction corresponding to any declination, between the limits, can be taken.

The corrections depending on declination have then been applied, with reversed signs, to the right ascensions of the stars used in the comparison, and the means of the residuals computed for each hour of right ascension. The result is shown in the second table.

The final tables give the corrections to the Hong Kong right ascensions, read off from the interpolating curves, which were drawn in the same manner for the corrections depending on right ascension and for those depending on declination. It will be understood that the complete reduction for any star is the sum of the reductions taken from these two tables, corresponding respectively to the declination and right ascension of the star.

Corrections depending on Declination.

Mean South Decl.	Computed Correction.	No. of Stars.	Mean South Decl.	Computed Correction.	No. of Stars.
°	s		°	s	
22·6	+·007	41	41·8	+·010	41
27·1	−·001	37	46·8	−·003	37
32·3	−·004	42	52·2	+·004	41
36·8	·000	37	56·8	−·004	24

Corrections depending on Right Ascension.

Mean R.A.	Computed Correction.	No. of Stars.	Mean R.A.	Computed Correction.	No. of Stars.
h m	s		h m	s	
0 27	−·014	13	5 28	−·008	13
1 32	−·006	13	6 31	+·006	14
2 30	+·005	13	7 29	+·005	15
3 30	−·002	12	8 27	−·015	12
4 22	−·003	11	9 32	+·006	13

Mean R.A. h m	Computed Correction. s	No. of Stars.	Mean R.A. h m	Computed Correction. s	No. of Stars.
10 32	−·001	10	17 28	+·013	12
11 31	−·008	11	18 29	+·005	12
12 26	+·009	12	19 33	−·006	16
13 30	+·003	14	20 34	+·004	12
14 28	+·009	9	21 22	−·009	11
15 39	−·009	9	22 24	+·021	14
16 30	+·001	4	23 26	−·004	15

Corrections depending on Declination.

South Decl. °	Adopted Correction. s	South Decl. °	Adopted Correction. s	South Decl. °	Adopted Correction. s
22·5	+·007	35·0	−·002	47·5	−·002
25·0	+·003	37·5	+·001	50·0	+·001
27·5	−·001	40·0	+·005	52·5	+·004
30·0	−·003	42·5	+·008	55·0	−·001
32·5	−·004	45·0	+·001	57·5	−·004

Corrections depending on Right Ascension.

R.A. h	Adopted Correction. s	R.A. h	Adopted Correction. s	R.A. h	Adopted Correction. s
0	−·010	8	−·007	16	−·004
1	−·010	9	−·007	17	+·007
2	−·002	10	+·003	18	+·009
3	+·002	11	−·003	19	−·001
4	−·003	12	·000	20	−·002
5	−·006	13	+·006	21	−·002
6	−·001	14	+·006	22	+·011
7	+·005	15	+·001	23	+·008

It should be noted that in the observations made at Hong Kong "stars of the 6th magnitude and brighter were screened, so as to appear of about magnitude $6\frac{1}{2}$ or 7." The stars included in the present comparison are, of course, for the most part bright stars, and would, therefore, in the great majority of cases have been observed through screens for the purpose of eliminating the relative magnitude-equation, as it affects the observed transits of bright stars.

1905 *April* 13.

Revised Elements of UY Cygni (Ch. 7514).

R.A. = $20^h\ 52^m\ 16^s$, Decl. = $+30°\ 2'·8$ (1900).

By A. Stanley Williams.

The period of $13^h\ 27^m\ 20^s·85$ found for this variable star in 1903 * was based on the assumption that a photograph obtained here on 1900 November 22 was taken when the star was either at, or at any rate very near, its maximum brightness. The observations of the summer and autumn of the same year showed, however, that this period was somewhat too short, and it appears that the star was really nearly an hour and a half past maximum at the time when the photograph above referred to was taken. Dr. E. Hartwig obtained a period of $13^h\ 27^m\ 27^s·59$ from the comparison of two maxima observed by him on 1902 June 28 and 1903 October 28.† In the present note fresh elements have been derived from the visual observations of the three years 1902, 1903, and 1904.

Table I. contains all the observed times of T_o, that is, the time when the variable in its rapid rise from minimum attained to equality with the comparison star *c* ($10^m·2$).‡ These observations were all made with a 6½-inch reflector. The third column of the table gives the observed geocentric time of T_o, and the fourth column the approximate correction for the light-equation. The other columns will explain themselves.

TABLE I.

Observed and Computed Times of T_o.

E.	Date.		Observed T_o. h m	Red to ⊙. m	Helio-centric T_o. h m	Computed T_o. h m	O−C. m
1138	1902 Aug.	22	10 10	+6·5	10 16·5	10 10·7	+ 5·8
1145		26	8 23	+6·5	8 29·5	8 22·7	+ 6·8
1147		27	11 18	+6·5	11 24·5	11 17·5	+ 7·0
1293	Nov.	17	7 52	+0·3	7 52·3	8 1·2	− 8·9
1634	1903 May	27	12 33	+0·7	12 33·7	12 52·4	−18·7
1789	Aug.	22	10 40	+6·5	10 46·5	10 43·0	+ 3·5
1814	Sept.	5	11 10	+6·3	11 16·3	11 8·6	+ 7·7
1821		9	9 18	+6·2	9 24·2	9 20·6	+ 3·6
1830		14	10 19	+6·0	10 25·0	10 27·3	− 2·3
1846		23	9 51	+5·5	9 56·5	9 46·1	+10·4
2408	1904 Aug.	3	12 35	+6·3	12 41·3	12 37·7	+ 3·6
2417		8	13 40	+6·4	13 46·4	13 44·5	+ 1·9
2422		11	8 43	+6·5	8 49·5	9 1·6	−12·1
2424		12	11 41	+6·5	11 47·5	11 56·5	− 9·0
2488	Sept.	17	9 18	+5·9	9 23·9	9 11·5	+12·4

* *Monthly Notices*, vol. lxiii. p. 307.

† *V.J.S. der Astron. Gesell.*, Jahrgang 38, Heft 3, p. 240.

‡ See diagram in the *Monthly Notices*, vol. lxiii. p. 305.

Table II. gives in similar form the observations of maximum. The two marked with an H in the last column were observed by Hartwig at Bamberg, the others by the writer. The photographic observation of 1900 November 22, alluded to above, has been added for comparison. As already remarked, it is nearly an hour and a half later than the actual time of maximum. Both the times of T_o in Table I. and those of maximum in Table II. were ascertained chiefly by means of single curves, though a mean light-curve was made use of in cases of uncertainty, or whenever this seemed desirable.*

TABLE II.

Observed and Computed Times of Maximum.

E.	Date.	Observed Maximum.	Red to ⊙.	Heliocentric Maximum.	Computed Maximum.	O−C.	
		h m	m	h m	h m	m	
0	1900 Nov. 22	[10 55]	−0·4	[10 54·6]	9 26·3	[+88·3]	p
1040	1902 June 28	...	...	12 30	12 46·1	−16·1	H
1072	July 16	11 40	+5·5	11 45·5	11 23·6	+21·9	
1138	Aug. 22	11 29	+6·5	11 35·5	11 33·5	+ 2·0	
1145	26	9 32	+6·5	9 38·5	9 45·5	− 7·0	
1147	27	12 29	+6·5	12 35·5	12 40·3	− 4·8	
1186	Sept. 18	9 34	+5·8	9 39·8	9 29·8	+10·0	
1293	Nov. 17	9 13	+0·3	9 13·3	9 24·0	−10·7	
1634	1903 May 27	13 53	+0·7	13 53·7	14 15·2	−21·5	
1641	31	12 31	+1·1	12 32·1	12 27·2	+ 4·9	
1789	Aug. 22	12 5	+6·5	12 11·5	12 5·8	+ 5·7	
1814	Sept. 5	12 19	+6·3	12 25·3	12 31·4	− 6·1	
1821	9	10 33	+6·2	10 39·2	10 43·4	− 4·2	
1830	14	11 38	+6·0	11 44·0	11 50·1	− 6·1	
1846	23	11 3	+5·5	11 8·5	11 8·9	− 0·4	
1908	Oct. 28	...	...	5 45·0	5 29·0	+16·0	H
2390	1904 July 24	11 42	+6·0	11 48·0	11 46·9	+ 1·1	
2408	Aug. 3	13 49	+6·3	13 55·3	14 0·5	− 5·2	
2417	8	15 1	+6·4	15 7·4	15 7·3	+ 0·1	
2422	11	10 19	+6·5	10 25·5	10 24·4	+ 1·1	
2424	12	13 23	+6·5	13 29·5	13 19·3	+10·2	
2463	Sept. 3	10 7	+6·3	10 13·3	10 8·8	+ 4·5	
2488	17	10 49	+5·9	10 54·9	10 34·3	+20·6	
2504	26	9 46	+5·3	9 51·3	9 53·1	− 1·8	

A period of $0^d·5607082$ was derived from the observations of T_o, and one of $0^d·5607125$ from the observations of maximum;

* All the times in the year 1902 were derived by means of a mean light-curve.

and giving the two results equal weight, the following are the revised elements of variation of *UY Cygni*:

$$\begin{aligned} T_0 &= 1900 \text{ November } 22,\ 8^h\ 3^m{\cdot}5 \text{ (G.M.T.)} \\ &\quad + 13^h\ 27^m\ 25^s{\cdot}37\ E \\ &= \text{J.D. } 2415346{\cdot}3358 + 0^d{\cdot}5607103\ E \\ \text{Maximum} &= 1900 \text{ November } 22,\ 9^h\ 26^m{\cdot}3 \text{ (G.M.T.)} \\ &\quad + 13^h\ 27^m\ 25^s{\cdot}37\ E \\ &= \text{J.D. } 2415346{\cdot}3933 + 0^d{\cdot}5607103\ E \end{aligned}$$

The times of T_0 and of maximum according to these elements will be found in the sixth columns of the foregoing tables, and the differences O—C in the last columns.

Note on the Variable RZ Lyræ.

R.A. = $18^h\ 39^m\ 54^s$, Decl. = $+32°\ 41'{\cdot}7$ (1900).

About 400 observations of the brightness of this star have been obtained during the past two years; but as the elements published in the *Ast. Nach.* 3880 still indicate the time of maximum within a few minutes, it does not seem necessary to discuss these at present. It may be mentioned, however, that the visual light-curve of the star corresponds very closely with the photographic light-curve published by Herr R. S. Dugan in the *Ast. Nach.* 3898.

Hove: 1905 *March* 22.

Revised Elements of Y Lyræ (*Ch.* 6685).

R.A. = $18^h\ 34^m\ 12^s$, Decl. = $+43°\ 51'{\cdot}8$ (1900).

By A. Stanley Williams.

The period of variation of this star published in the *Monthly Notices* for January 1902 was derived solely from the comparison of three photographs. Two of these, dated respectively 1899 December 31 and 1900 September 3, had evidently been taken when the star was nearly at its maximum brightness, and in November 1901 several photographs were taken near maximum, one of which was selected as seeming to show practically the same phase as the other two. Although the resulting period of $12^h\ 3^m\ 52^s{\cdot}21$ still satisfies closely the observations, yet it now seems desirable to derive fresh elements of variation from the visual observations alone, since the latter now extend over a period of four years.

Table I. contains all the observed times of T_0, that is, the

time when the variable in its rapid rise from minimum attained to equality with the comparison star *l* (12^{m}·1). These observations were all made by the writer with a 6½-inch reflector, excepting that of 1901 October 25, which is derived from the observations by Dr. E. Hartwig at Bamberg, published in the note at the foot of page 205 of vol. lxii. of the *Monthly Notices*. The different columns of the table will sufficiently explain themselves. The approximate corrections for the equation of light have been applied, although this correction is never very large in the case of *Y Lyræ*.

TABLE I.

Observed and Computed Times of T_o.

E.	Date.	Observed T_o.	Red to ☉.	Helioc T_o.	Computed T_o.	O−C.	
	1901	h m	m	h m	h m	m	
1184	Aug. 18	10 22	+2·4	10 24·4	10 11·0	+13·4	
1186	19	10 10	+2·3	10 12·3	10 18·8	− 6·5	
1188	20	10 10	+2·3	10 12·3	10 26·5	−14·2	
1190	21	10 30	+2·3	10 32·3	10 34·3	− 2·0	
1192	22	10 40	+2·2	10 42·2	10 42·1	+ 0·1	
1194	23	10 41	+2·2	10 43·2	10 49·9	− 6·7	
1196	24	10 50	+2·2	10 52·2	10 57·6	− 5·4	
1200	26	11 11	+2·1	11 13·1	11 13·1	0·0	
1218	Sept. 4	12 7	+1·7	12 8·7	12 22·9	−16·2	
1309	Oct. 20	6 11	−0·7	6 10·3	6 15·8	− 5·5	
1319	25	7 2	−1·0	7 1·0	6 54·7	+ 6·3	H
1337	Nov. 3	8 12	−1·5	8 10·5	8 4·6	+ 5·9	
	1902.						
1757	June 2	11 13	+2·7	11 15·7	11 13·6	+ 2·1	
1918	Aug. 22	9 30	+2·2	9 32·2	9 38·2	− 6·0	
1950	Sept. 7	11 49	+1·6	11 50·6	11 42·3	+ 8·3	
1952	8	11 52	+1·6	11 53·6	11 50·1	+ 3·5	
	1903.						
2521	June 21	12 37	+2·7	12 39·7	12 37·2	+ 2·5	
	1904.						
3231	June 12	10 35	+2·9	10 37·9	10 31·2	+ 6·7	
3243	18	11 20	+3·0	11 23·0	11 17·8	+ 5·2	
3400	Sept. 5	9 22	+1·6	9 23·6	9 26·8	− 3·2	
3408	9	9 50	+1·5	9 51·5	9 57·7	− 6·2	
3410	10	10 3	+1·5	10 4·5	10 5·5	− 1·0	

Table II. gives in similar form the observed times of maximum. Those marked with H in the last column were observed by Hartwig at Bamberg,* the others by the writer. The three observations marked with a "p" are the three

* *V.J.S. der Astron. Gesell.*, Jahrgang 36, Heft 3/4, p. 268.

photographic observations previously alluded to. They were not used in the calculations, and have been inserted here merely for the sake of comparison.

TABLE II.

Observed and Computed Times of Maximum.

E.	Date.	Observed Max. h m	Red to ⊙. m	Helioc. Max. h m	Computed Max. h m	O−C.	
	1899.						
0	Dec. 31	[6 43·5]	−3·1	[6 40·4]	6 35·3	[+ 5·1]	p
	1900.						
490	Sept. 3	[14 17·5]	+1·8	[14 19·3]	14 16·0	[+ 3·3]	p
	1901.						
1184	Aug. 18	[11 51]	+2·4	[11 53·4]	11 7·9	[+45·5]	
1186	19	11 32	+2·3	11 34·3	11 15·7	+18·6	
1188	20	11 34	+2·3	11 36·3	11 23·4	+12·9	
1190	21	11 33	+2·3	11 35·3	11 31·2	+ 4·1	
1192	22	11 33	+2·2	11 35·2	11 39·0	− 3·8	
1194	23	11 45	+2·2	11 47·2	11 46·8	+ 0·4	
1196	24	12 13	+2·2	12 15·2	11 54·5	+20·7	
1200	26	12 11	+2·1	12 13·1	12 10·0	+ 3·1	
1218	Sept. 4	13 32	+1·7	13 33·7	13 19·8	+13·9	
1309	Oct. 20	7 10	−0·7	7 9·3	7 12·7	− 3·4	
1323	27	8 0	−1·1	7 58·9	8 7·2	− 8·3	H
1331	31	8 22	−1·3	8 20·7	8 38·1	−17·4	H
1333	Nov. 1	8 24	−1·4	8 22·6	8 45·9	−23·3	H
1335	2	8 32	−1·4	8 30·6	8 53·7	−23·1	H
1337	3	8 34	−1·5	8 32·5	9 1·5	−29·0	H
1337	3	9 12	−1·5	9 10·5	9 1·5	+ 9·0	
1339	5	[9 11]	−1·6	[9 9·4]	9 9·2	[+ 0·2]	p
	1902.						
1757	June 2	12 7	+2·7	12 9·7	12 10·5	− 0·8	
1918	Aug. 22	10 42	+2·2	10 44·2	10 35·1	+ 9·1	
1950	Sept. 7	12 43	+1·6	12 44·6	12 39·2	+ 5·4	
1952	8	12 51	+1·6	12 52·6	12 47·0	+ 5·6	
	1904.						
3243	June 18	12 3	+3·0	12 6·0	12 14·7	− 8·7	
3384	Aug. 28	9 10	+2·0	9 12·0	9 21·6	− 9·6	
3386	29	9 40	+1·9	9 41·9	9 29·4	+12·5	
3396	Sept. 3	10 4	+1·7	10 5·7	10 8·1	− 2·4	
3400	5	10 13	+1·6	10 14·6	10 23·7	− 9·1	
3408	9	10 49	+1·5	10 50·5	10 54·6	− 4·1	
3410	10	11 6	+1·5	11 7·5	11 2·4	+ 5·1	

From the observations of T_o a period of $0^d{\cdot}5026941$ was derived, and from those of the maximum one of $0^d{\cdot}5026930$.

Giving the former result double weight, the following are the revised elements of variation of *Y Lyræ*:

$$\begin{aligned} T_o &= 1899 \text{ Dec. } 31,\ 5^h\ 38^m{\cdot}4\ (\text{G.M.T.}) + 12^h\ 3^m\ 52^s{\cdot}74\ E \\ &= \text{J.D. } 2415020{\cdot}2350 + 0^d{\cdot}5026937\ E \\ \text{Maximum} &= 1899 \text{ Dec. } 31,\ 6^h\ 35^m{\cdot}3\ (\text{G.M.T.}) + 12^h\ 3^m\ 52^s{\cdot}74\ E \\ &= \text{J.D. } 2415020{\cdot}2745 + 0^d{\cdot}5026937\ E \end{aligned}$$

The computed times of T_o and of maximum will be found in the sixth columns of the foregoing tables, and the differences O—C in the last columns. With reference to these latter it should be noted that Hartwig seems to have systematically observed the maxima earlier than the writer, and that the grouping of the residuals is suggestive of the existence of subjective influences. In particular the late maximum of 1901 August 18, the first visually observed, is probably due to a struggle on the part of the observer against the rapid decline in brightness after maximum. There are no observations of maximum in 1903 and only one of T_o. *Y Lyræ* is rather a faint variable for observation with a $6\frac{1}{2}$-inch aperture, and a very clear night and absence of moonlight are essential in order to obtain a satisfactory series of observations. In the year above mentioned unfortunately all the clear and cloudless nights, when the maxima were observable, occurred when there was a bright moon. All the times of T_o and of maximum given in the preceding tables and observed here were derived from single curves, though, as regards the maxima at any rate, a somewhat greater accordance might have been obtained by the use of a mean light-curve.

It should be mentioned that a slightly different light-scale was used in reducing the observations of 1902–4. In 1901 the observations had been made with the eyes kept normal to a line joining the stars A and *b* (see the diagram in the *Monthly Notices*, vol. lxii. p. 201), but in the subsequent years they were made with the head so held that the eyes were parallel to a straight line joining the two stars undergoing comparison. The following is the light-scale used in reducing the observations of 1902–4, with the assumed magnitudes of the comparison stars :

Star.	Light-scale.	Mag.
d	38·4	10·11
c	29·9	10·70
h	15·8	11·69
l	10·0	12·10

It does not seem likely that any sensible difference will have been caused in the time of T_o by the change of scale, the position of the comparison star relative to the minimum brightness of the variable being nearly the same in both scales.

Hove: 1905 *March* 9.

Value of Meteoric Radiants based on Three Paths.

By W. F. Denning.

With regard to the validity of radiants derived from only three paths, discussed mathematically by Mr. H. W. Chapman (*Monthly Notices*, 1905 January, p. 238), I would like to offer a few remarks from an observer's point of view.

1. I believe that, as a rule, radiants determined from three paths are useless or at least extremely doubtful, especially when the observers responsible for them have not gained considerable experience in meteoric work.

2. There are particular cases, however, where three meteors may indicate good radiants, and these are when the latter are low in altitude or very near the horizon. In such instances the meteors traverse long paths; they have comparatively slow motions ascending, and the discriminating observer can entertain no doubt as to the place of divergence. On the other hand, the greatest uncertainty must attach to short, swift meteors falling almost vertically in low positions. When the flights of these are carried back in the same lines they may each cross a dozen or more active radiants.

3. Under ordinary circumstances it is far from safe to accept centres resting upon less than five paths. If very meagrely supported positions are included in our catalogues and accorded equal weight with others well determined from ten or twenty tracks it will be impossible to get good mean positions, and the records of this branch will soon be burdened with many non-existent showers occasioning doubts and complications very difficult to eliminate.

4. As there are a multitude of showers distributed over the sky some of the radiants lie near together, and there must naturally occur a large number of accidental intersections of three meteors forming false appearances of radiation.

5. Of course much depends upon the experience, judgment, and method of the observer. Those who rely only upon the directions of flight will frequently deduce false positions, as this is an insufficient criterion. Others who carefully sort the meteors according to their individual velocities, streaks, trains, &c., and also consider the length of path and position of the assumed radiant relatively to the horizon, will seldom err.

6. There are very marked differences in the capacity of individual observers. Considerable practice will not make every observer reliable and accurate. Some workers in this department will generally ascertain radiants within 1° or 2° of probable error, while others will ordinarily make mistakes of 5° or 10°, and fictitious radiants will occur so frequently in the results of the latter as practically to negative their value.

7. In estimating the validity of radiants resting on very few tracks it is, therefore, most important to consider the authority

for these positions and the conditions affecting the determination in each case.

8. As a partial safeguard against pseudo-radiants I would suggest that no radiant be accepted depending upon less than five paths *unless the circumstances are special.* It is true that certain meteoric systems—in fact a large majority of them—are so tenuous that to gather five paths from any of these positions may necessitate forty or fifty hours of watching by one observer, and that many such showers must altogether escape detection if the limit of a proved radiant be placed at five members. But I feel sure that it will be best in the end to reject all very slenderly supported showers *indicated under ordinary conditions.* Even the most proficient observers are sometimes led astray by insufficient materials, and meteoric streams are so abundant that it will be safer to recognise such results only as can be fairly well substantiated by adequate data.

9. Exception might occasionally be made in cases where the observer has acquired extensive experience, and where he has accurately registered three or four meteors under circumstances specially favouring the detection of their radiant.

10. Observers should keep a list of feebly suspected showers met with during the progress of observation and endeavour to corroborate them at similar epochs in following years. By combining materials in this manner good and amply supported radiants may often be obtained.

11. I have sometimes secured certain evidence of radiants from only two, three, or four tracks when the positions have been near the horizon. Thus on 1878 July 28 only two paths served to define a centre at 33°—20° in the eastern region of *Cetus* :—

	G.M.T. h m	Mag.	From ° °	To ° °	Length. °
July 28	13 32	1	2 + 0	336 + 17	31
„	14 32	2	55 + 44	126 + 74	41

In 1886 April–May I recorded three meteors from a radiant in *Scorpio* at about 254°–20° :—

	h m	Mag.	From ° °	To ° °	Length. °
April 30	11 39	4	$249\frac{3}{4} + 34$	240 + 76	36
May 5	14 29	2	$287\frac{1}{2} - 5$	$315 + 9\frac{1}{2}$	31
May 6	10 37	2	$233\frac{1}{2} + 26\frac{1}{2}$	202 + 57	38

Bishopston, Bristol :
1905 *March* 18.

On the Spherical Correction of Object-glasses.

By A. E. Conrady.

The performance of object-glasses and mirrors is influenced very considerably by unavoidable residuals of spherical aberration. The usual treatment of these residuals, and of spherical aberration in general, by purely geometrical methods, *i.e.* as causes of a more or less serious confusion of the "rays" forming the image, is decidedly unsatisfactory from the physical point of view, for the undulatory theory defines a perfect lens or mirror as one which brings all light from a luminous point to the focus *in the same phase of vibration*, or which, in other words, transforms the spherical waves sent out by the object into truly spherical waves converging towards the focus. From this point of view any defects in a lens or mirror can become sensible only in so far as they sensibly throw certain portions of the total light *out of phase* with other portions ; and there can be no doubt that the differences of phase with which the different portions of the total light meet at the focus are the only true and absolute measure of the seriousness of any outstanding aberrations.

A considerable amount of knowledge of spherical aberration in this physical sense may be derived from the known peculiarities in the course of the "rays" of geometrical optics when we bear in mind that the latter are properly defined as normals of the wave surface.

Lenses which combine small angular aperture with small dimensions can frequently be so well corrected that either trigonometrical computation or experimental test shows no sensible departure of any ray from the desired direction towards the focus ; and as the sphere is the only surface which has all its normals intersecting in one point, it follows, as a matter of course, that in such cases the physical requirement as to phase-relation is strictly fulfilled.

But in the majority of cases, and always in lens-systems of either wide angular aperture (microscope-objectives) or of large dimensions (telescope-objectives) it is impossible completely to free all zones of the system from spherical aberration in the usual geometrical sense. Such lenses are said to be affected with "spherical zones," and in such cases the phase-relations at the focus remain to be investigated.

In the simplest case, *i.e.* that of object glasses of the type usually found in telescopes, computation shows that when spherical correction is established in the usual way by uniting the paraxial and the marginal rays, all intermediate zones are "undercorrected," or cut the axis nearer the lens. From this and from the above definition of rays we can conclude that the extra-axial parts of waves refracted by such systems are in advance of the ideal spherical waves converging towards the

common focus of the axial and marginal rays ; for it will be seen by reference to fig. 1 that only a generating line like that drawn can produce a surface with an arrangement of normals such as the geometrical theory proves to exist.

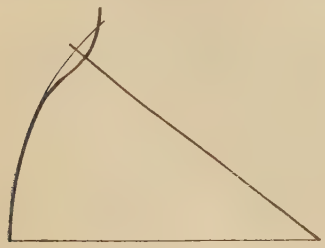


Fig. 1.

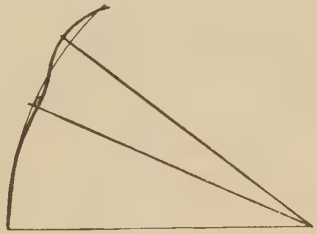


Fig. 2*a*.

In the case of lens-systems which contain deeply curved lenses, and in which the spherical residuals might attain prohibitive magnitude, the correction is sometimes carried a step farther by uniting the paraxial and the marginal rays with those passing through some selected intermediate zone. Further computation then again shows that all other zones are defective, and we can estimate that the real wave-form must be of the nature of those shown in fig. 2*a*, *b*, *c*, ; that it may indeed be any surface of rotation having the centre of curvature of its paraxial

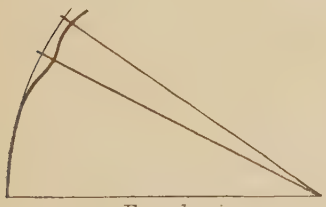


Fig. 2*b*.

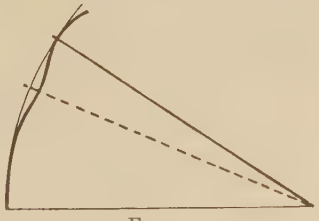


Fig. 2*c*.

portion in the computed focus, and having two other zones the normals of which are directed towards the same focus. The most interesting result of the above simple and elementary considerations is probably this, that the zones for which spherical correction in the geometrical sense is established are apt to be the worst corrected in the physical sense. On the other hand the same considerations prove the desirability of practicable methods by which the *magnitude* of the existing differences of phase may be determined.

I say *practicable* methods because it would be a simple trigonometrical task to compute the *total* path of a number of rays through any lens-system ; indeed, it will be seen by reference

to my paper on chromatic aberration* that the quantities D there used and trigonometrically defined represent successive portions of such paths ; but as the total path is measured by the foot-rule and the wave-length by tenth-meters, it might occasionally require the hunting-up of a table of 10-figure logarithms to attain the necessary accuracy.

The formulæ to be sought for should therefore give the *difference* between different optical paths in terms which are reasonably small multiples of the wave-length.

The first method fulfilling this condition which I succeeded in devising more than ten years ago is briefly this :

Let f O f' (fig. 3) be the optical axis of some lens-system ; O P a refracting surface, f at the distance l from O a luminous point in the medium of refractive index n, to the left of the

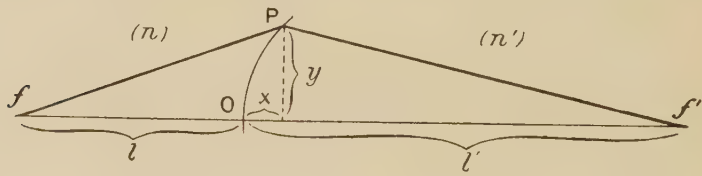


Fig. 3.

refracting surface ; f' at the distance l' from O and in the medium of index n' the point for which we seek the difference of the optical paths of the axial ray f O f' and the marginal ray f P f', the point P being defined by coordinates x and y.

The coordinates shall be positive to the right of O and above the axis $f f'$, negative to the left of O and below $f f'$

Bearing this in mind, we deduce by inspection

$$\text{Path of axial ray} \quad = l'-l$$

$$\text{Path of marginal ray} = \sqrt{(l'-x)^2+y^2}-\sqrt{(l-x)^2+y^2}$$

To obtain the *optical* paths we must multiply any distance by the index of the medium in which it is measured in order to reduce it to its *equivalent* distance in space ; for the refractive index is inversely proportional to the velocity of light in any medium.

With this definition we can write down the difference between the optical paths of axial minus marginal rays as

$$\text{I.} \quad w = n'(l'-\sqrt{(l'-x)^2+y^2})-n(l-\sqrt{(l-x)^2+y^2})$$

As we have made no restrictions either with regard to the refracting surface or the points f and f', this equation is quite general. If we put $w = 0$ it is the general equation of the

* *Monthly Notices*, 1904 January, p. 186.

generating line of a refracting surface which is aplanatic with regard to the conjugates f and f'' (Cartesian ovals).

We will now restrict ourselves to a spherical surface of radius r; this gives us the well-known relation

$$y^2 = 2rx - x^2$$

between x and y. Introducing this for y^2 in I., and carrying out a few other obvious operations at the same time, we obtain

$$\text{I.*}\quad w = n'l'\left(1 - \sqrt{1 - 2\frac{x}{l'}\frac{l'-r}{l'}}\right) - nl\left(1 - \sqrt{1 - 2\frac{x}{l}\frac{l-r}{l}}\right)$$

putting

$$c' = \frac{x}{l'}\frac{l'-r}{l'} \qquad c = \frac{x}{l}\frac{l-r}{l}$$

and developing the square roots by McLaurin's theorem, we find

$$w = \left\{\begin{matrix} n'l'(c' + \tfrac{1}{2}c'^2 + \tfrac{1}{2}c'^3 + \tfrac{5}{8}c'^4 + \tfrac{7}{8}c'^5 + \quad \dots \quad \dots \quad \dots) \\ -nl(c + \tfrac{1}{2}c^2 + \tfrac{1}{2}c^3 + \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots) \end{matrix}\right\}$$

or separating the first-order terms

$$\text{II.}\quad w = n'l'c' - n.l.c + \left\{\begin{matrix} n'l'c'^2(\tfrac{1}{2} + \tfrac{1}{2}c' + \tfrac{5}{8}c'^2 + \quad \dots \quad \dots) \\ -n.l.c^2(\tfrac{1}{2} + \tfrac{1}{2}c + \tfrac{5}{8}c^2 + \quad \dots \quad \dots) \end{matrix}\right\}$$

The terms c' and c involve the abscissa x which in the case of a circle diminishes rapidly as P approaches the axis; hence for paraxial rays the first terms of II. will alone be sensible, giving ${}_0w = n'l'c' - nlc$.

If now we put ${}_0w = 0$ we obviously have the condition which makes f' the focus of the paraxial rays from f, for ${}_0w = 0$ means that all light passing from f through the paraxial zone arrives simultaneously at f' or becomes a spherical wave-train converging towards f'; and adopting this further restriction as to the points f and f' being paraxial conjugates, we have l' determined by the condition ${}_0w = 0$; for reintroducing the values of c and c' we obtain the condition

$$n'\frac{l'-r}{l'} = n\frac{l-r}{l} \text{ or } \frac{n'}{l'} = \frac{n}{l} + \frac{n'-n}{r}$$

which is arrived at in geometrical optics by a totally different method.

We now have for the focus of the paraxial rays thus determined

$$w = n'l'c'^2(\tfrac{1}{2} + \tfrac{1}{2}c' + \tfrac{5}{8}c'^2 + \dots) - nlc(\tfrac{1}{2} + \tfrac{1}{2}c + \tfrac{5}{8}c^2 + \dots)$$

The rigorous numerical computation of this would complete our task. And this is *possible* because the c and c' are, in all practical cases, limited to a narrow range of values, from about

$-1{\cdot}5$ to $+{\cdot}3$; hence we can prepare a table * giving the numerical value of the round brackets for any practically possible c, really a very short and easily computed table. The series itself can be used for the small values of c usually occurring, whilst for the larger values it is easy to see, by referring back to the original equation, that we have

$$f_{(c)} = \tfrac{1}{2} + \tfrac{1}{2}c + \tfrac{5}{8}c^2 + \ldots = \frac{1 - \sqrt{1-2c} - c}{c^2}$$

and that the value of the series may be obtained from this formula. Calling this tabulated value of the series $f_{(c)}$ we then have the simple formula for the difference of optical paths

III. $$w = n'l'c'^2 f_{(c')} - nlc^2 f_{(c)}$$

Up to this point the solution of the problem is a very simple and convenient one ; but when applying it to a succession of refracting or reflecting surfaces, a troublesome correction has to be taken into account which arises from the presence of spherical aberration in the geometrical sense.

Equation III. is rigorously correct for rays directed towards the focus of the paraxial pencil, but as a rule the marginal rays do not travel in that direction, being deflected through spherical

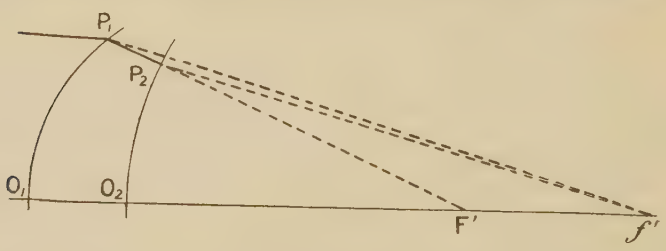


FIG. 4.

aberration ; and thence arises the correction. Referring to fig. 4, where f' is assumed to be the focus of the paraxial pencil

* The following values of $\log f_{(c)}$ may be welcome to those who would like to compute by this process :

c	$\log f_{(c)}$	c	$\log f_{(c)}$
$-{\cdot}10$	$9{\cdot}6585$	$+{\cdot}00$	$9{\cdot}6990$
·09	9·6623	+·01	9·7033
·08	9·6661	·02	9·7078
·07	9·6701	·03	9·7123
·06	9·6740	·04	9·7169
·05	9·6780	·05	9·7215
·04	9·6821	·06	9·7263
·03	9·6862	·07	9·7311
·02	9·6904	·08	9·7360
·01	9·6947	·09	9·7410
		·10	9·7461

as refracted by the surface O_1P_1, F' the corresponding focus of the marginal rays refracted by the same surface, Equation III. assumes the light to proceed along the straight line P_1f'; according to the law of refraction the light takes the path P_1F', reaching the second surface at P_2.

Without entering into an extended discussion, it may be merely stated that the correction is found to be equal to the excess of the two sides $(P_1P_2+P_2f')$ of the long triangle P_1P_2f' over the third side P_1f', and that the computation of this correction was often found to be the most troublesome part of the task. Still, in the case of ordinary telescope-objectives of the usual proportion of aperture to focus, the correction is almost insensible, and may safely be neglected; and in this case Equation III. may therefore be regarded as a complete and convenient solution of the problem. It is different, however, in lens-systems such as are found in microscope-objectives. Here the correction occasionally amounts to many wave-lengths, and becomes an essential part of the whole difference of optical paths.

It only remains to be added that by substituting $(n+dn)$ for n and $(n'+dn')$ for n' in Equation I. and reducing in a manner analogous to that applied to I. an expression may be obtained which allows of the correction of chromatic aberration in a manner analogous to that treated of in my paper on chromatic correction. As the method given in that paper is more convenient, I will not give the modification in detail.

As the troublesome correction mentioned above arises because the marginal rays are not directed towards the focus of the paraxial rays which is the point of reference in Equation III., this correction would obviously disappear if the point of intersection of the marginal rays with the optical axis were taken instead.

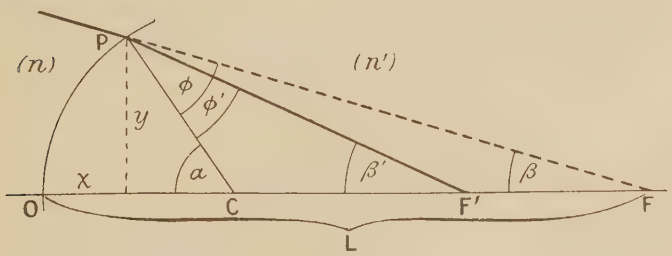


Fig. 5.

Realising this, I endeavoured for years to find a convenient solution of the problem on this basis, but until recently without success. As the path of a marginal ray can be conveniently defined and computed only by trigonometrical equations, it was obvious that the solution would have to be a trigonometrical one also.

As is often the case, the finding of the elegant formula which follows depended on the choice of suitable variables and a "lucky" transformation.

The trigonometrical formulæ by which the course of rays through optical systems is defined were given in the former paper repeatedly quoted; referring to fig. 5 here reproduced, rays are defined by the abscissa $OF = L$ and the angle of convergence β, and if the coordinates of the refracted ray are distinguished by a dash the relations between the quantities are

$$1. \quad \sin\phi = \sin\beta\frac{L-r}{r}$$

$$2. \quad \alpha = \beta+\phi$$

$$3. \quad n'\sin\phi' = n\sin\phi$$

$$4. \quad \beta' = \alpha-\phi'$$

$$5. \quad L'-r = r\frac{\sin\phi'}{\sin\beta'}$$

We assume that the course of the rays to be considered has been computed by these equations, and that the quantities contained in them are therefore known.

Then we can obtain the difference of the optical paths of the marginal ray and of that proceeding along the optical axis which is produced by the refraction in the following manner.

Let a spherical wave, OA (fig. 6) in the medium of index n to the left of the refracting spherical surface OP be converging towards F; while the marginal portion is traversing the distance AP in the medium of index n, the axial part has entered the medium of index n' to the right of the refracting surface, but the *optical* paths of the two portions are necessarily the same. As the marginal ray is refracted towards F′ we should, on the supposition of freedom from spherical aberration, have a new

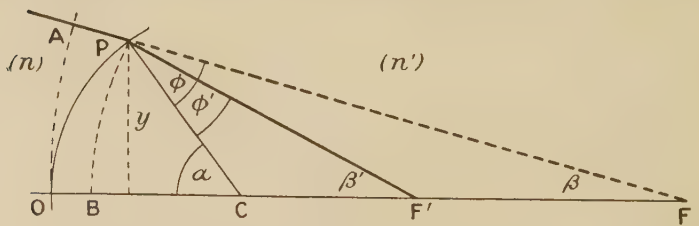


Fig. 6.

spherical wave, BP, with F′ as centre, and the axial ray *should* have reached B in the second medium at the instant when the marginal ray reaches the refracting surface at P; the optical path OB on the axis should therefore be equal to that AP of

the marginal ray ; hence if these two optical paths are separately computed and found to differ, the difference is due to, and is a measure of, the spherical aberration produced by the surface OP, and is therefore the quantity we are trying to determine.

To obtain an expression for the length of AP we transfer it to the optical axis by a circle, PP′, with F as centre, so that

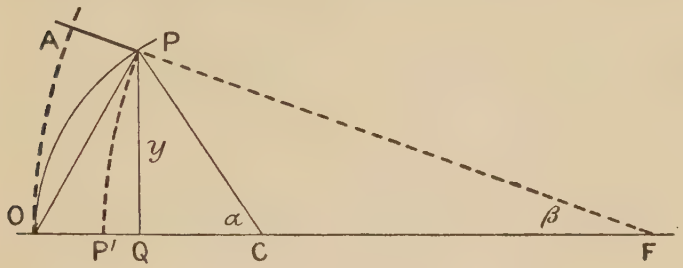


Fig. 7.

OP′ = AP (fig. 7). Dropping a perpendicular y from P upon the optical axis, we see that OP′ = OQ−P′Q.

Drawing OP, we have in the isosceles triangle OCP the angle at O = $90° - \frac{\alpha}{2}$; hence in the right-angled triangle OQP

$$OQ = y \tan \frac{\alpha}{2}$$

Similarly we find

$$P'Q = y \tan \frac{\beta}{2}$$

hence

$$OP' = AP = y \left(\tan \frac{\alpha}{2} - \tan \frac{\beta}{2} \right)$$

Applying the same process in the determination of the length of OB we find

$$OB = y \left(\tan \frac{\alpha}{2} - \tan \frac{\beta'}{2} \right)$$

Multiplying the two paths by the respective refractive indices, we obtain the optical paths ; their difference W is therefore

$$\text{IV.} \quad W = n'y \left(\tan \frac{\alpha}{2} - \tan \frac{\beta'}{2} \right) - ny \left(\tan \frac{\alpha}{2} - \tan \frac{\beta}{2} \right)$$

This equation may be further reduced in a few steps by the following transformations :

Replacing tan by $\frac{\sin}{\cos}$ in Equation IV. we obtain by cross-multiplication

$$W = n'y\frac{\sin\frac{\alpha-\beta'}{2}}{\cos\frac{\alpha}{2}\cos\frac{\beta'}{2}} - ny\frac{\sin\frac{\alpha-\beta}{2}}{\cos\frac{\alpha}{2}\cos\frac{\beta}{2}}$$

by (4) $\frac{\alpha-\beta'}{2} = \frac{\phi'}{2}$ by (2) $\frac{\alpha-\beta}{2} = \frac{\phi}{2}$

hence

$$W = n'y\frac{\sin\frac{\phi'}{2}}{\cos\frac{\alpha}{2}\cos\frac{\beta'}{2}} - ny\frac{\sin\frac{\phi}{2}}{\cos\frac{\alpha}{2}\cos\frac{\beta}{2}}$$

Multiplying the first term by $\frac{2\cos\frac{\phi'}{2}}{2\cos\frac{\phi'}{2}}$, the second by $\frac{2\cos\frac{\phi}{2}}{2\cos\frac{\phi}{2}}$, and applying the general formula $2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = \sin\alpha$ we obtain

$$W = \frac{n'y\sin\phi'}{2\cos\frac{\alpha}{2}\cos\frac{\beta'}{2}\cos\frac{\phi'}{2}} - \frac{ny\sin\phi}{2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\phi}{2}}$$

and as by (3) $n'\sin\phi' = n\sin\phi$ we have

$$W = \frac{n'y\sin\phi'}{2\cos\frac{\alpha}{2}}\left\{\frac{1}{\cos\frac{\beta'}{2}\cos\frac{\phi'}{2}} - \frac{1}{\cos\frac{\beta}{2}\cos\frac{\phi}{2}}\right\}$$

or

$$W = \frac{n'y\sin\phi'}{2\cos\frac{\alpha}{2}} \cdot \frac{\cos\frac{\beta}{2}\cos\frac{\phi}{2} - \cos\frac{\beta'}{2}\cos\frac{\phi'}{2}}{\cos\frac{\beta}{2}\cos\frac{\beta'}{2}\cos\frac{\phi}{2}\cos\frac{\phi'}{2}}$$

In the *numerator* of this we introduce $\cos\frac{\beta}{2} = \cos\left(\frac{\beta-\beta'}{2} + \frac{\beta'}{2}\right)$, an obvious identity, and resolve this by the usual formula. Similarly we introduce $\cos\frac{\phi'}{2} = \cos\left(\frac{\beta-\beta'}{2} + \frac{\phi}{2}\right)$, which follows by combining (2) and (4), which give $\beta+\phi = \beta'+\phi' = \alpha$, and resolve this also. We then obtain the numerator of the second fraction in the last equation for W in the form of four terms,

two of which cross out, whilst the other two combine, with this result :

$$\text{V.} \qquad W = \frac{n' \cdot y \cdot \sin\phi' \sin\frac{\beta-\beta'}{2} \sin\frac{\phi-\beta'}{2}}{2\cos\frac{a}{2}\cos\frac{\beta}{2}\cos\frac{\phi}{2}\cos\frac{\beta'}{2}\cos\frac{\phi'}{2}}$$

This remarkable equation is undoubtedly the simplest and clearest *rigorous* expression to which the problem of spherical aberration has ever been reduced, and its discussion is therefore as interesting as it is easy.

As the angles occurring in V. are usually small, we see that the important terms are those of the numerator, the denominator being more in the nature of a correction.

If we now discuss the effect of variation of any of the five terms in the numerator, the first, n', tells us that, other things being equal, the aberration is proportional to the refractive index. The second term, similarly, makes the aberration proportional to the semi-aperture y ; hence we can at once generalise that in similarly constructed optical systems the residual aberrations are proportional to the scale to which the different systems are constructed, and therefore that residuals which would be immaterial in a small object-glass may easily become serious in a similar object-glass of large size.

The term $\sin\phi'$ next shows that the aberration is approximately proportional to the angle of refraction, and the following term that it is also nearly proportional to the deflection produced by the refraction ; these are both results which seem almost foregone conclusions, and call for no special notice. But now we come to the last term, and in this are wrapped up some of the chief mysteries of object-glass construction ; for this term can become zero when all the other quantities have finite values, and when the course of the rays is therefore altered by refraction ; it thus expresses the existence of points for which the sphere is free from spherical aberration.

Let us discover these points.

$\frac{\phi-\beta'}{2}$ becomes zero when $\phi = \beta'$, which implies $\sin\phi = \sin\beta'$, and, further, by (3) $\frac{n'}{n}\sin\phi' = \sin\beta'$. The vanishing of the last term of V. therefore depends on the condition $\frac{\sin\beta'}{\sin\phi'} = \frac{n'}{n}$.

If we now refer again to fig. 5, the trigonometrical sine-law applied to the triangle PCF′ gives us

$$\frac{\sin\beta'}{\sin\phi'} = \frac{r}{L'-r} = \frac{n'}{n}, \text{ or } L'-r = r\frac{n}{n'}, \text{ or (6) } L' = r\frac{n'+n}{n'}$$

Further, by (2) and (4) we have $\phi-\beta' = \phi'-\beta$, or, proceeding as before,

$$\sin\phi' = \sin\beta = \frac{n}{n'}\sin\phi, \text{ or } \frac{\sin\beta}{\sin\phi} = \frac{n}{n'}$$

whence, by the triangle PCF,

$$\frac{\sin\beta}{\sin\phi} = \frac{n}{n'} = \frac{r}{L-r};\ L-r = r\frac{n'}{n};\ (7)\ L = r\frac{n'+n}{n}$$

whilst $\sin\phi = \sin\beta'$, combined with $\frac{n}{n'}\sin\phi = \sin\beta$, both equations occurring in the above, give

$$(8)\ \frac{\sin\beta}{\sin\beta'} = \frac{n}{n'} = \text{a constant},$$

to which we may add, by combining (6), (7), and (8):

$$(8^*)\ \frac{L'}{L} = \frac{n}{n'} = \frac{\sin\beta}{\sin\beta'}$$

By (6) and (7) the position of the pair of aplanatic points is defined. Equation 8 gives the important further information that these are aplanatic points in the wider sense of Professor Abbe, for they fulfil the optical sine-condition. The construction of wide-angled microscope-objectives depends on this property of the sphere, for only by taking advantage of these aplanatic points is it possible to deal with cones of light of as much as 140° angular aperture without introducing overwhelming amounts of spherical aberration.

On the other hand we now see that when ϕ and β have opposite signs the last term in V. tends to make the spherical aberration particularly heavy; this also is taken advantage of in object-glass construction when it is desired to obtain a large amount of spherical aberration—generally in the sense of *over*-correction—with only a moderate alteration of convergence. This discussion might be carried much further, but it may here suffice to add that if the rays converge towards points nearer the centre of curvature of the refracting surface than those defined by (6) and (7) above, we obtain refraction in the sense of that produced by a convex lens accompanied by spherical aberration in the sense of that of an ordinary concave lens (over-correction), a little known fact.

Equation V. has been deduced on the assumption of a spherical impinging wave; it will, however, be clear that any retardation or acceleration of the arriving marginal ray would be simply added to that produced by the refraction, and, further, that the total difference of optical paths of a marginal and the axial ray in any optical system will be the algebraical sum of all differences arising at successive surfaces.

Referring once more to fig. 1 it will be seen that these new equations make it possible to compute the distance between the real refracted wave and the ideal one when spherical correction in the geometrical sense has been established and thus to obtain a trustworthy indication of the residual aberration; but an even more important application of the new equations when so used in conjunction with the usual trigonometrical computation lies in the higher correction of spherical aberration by adding to the usual geometrical condition, that the lens-system shall be so corrected that the marginal rays shall be directed towards the focus of the paraxial pencil, the further one *that the marginal rays shall also meet the paraxial ones without any difference of optical paths,* i.e. *in the same phase of vibration.*

This condition, which I believe to be quite original with me, is one which I have adopted as a definite standard in carrying spherical correction to higher perfection in all cases which call for it.

Equation IV. gives the difference between the marginal and axial paths; it may therefore be differentiated to find its variation with the wave-length—in other words the chromatic aberration—after the manner introduced by me in the paper quoted above. We obtain

$$\text{IV.*}\quad \frac{\partial W}{\partial \lambda} = \frac{\partial n'}{\partial \lambda} \cdot y\left(\tan\frac{\alpha}{2} - \tan\frac{\beta'}{2}\right) - \frac{\partial n}{\partial \lambda} y\left(\tan\frac{\alpha}{2} - \tan\frac{\beta}{2}\right)$$

Really α, β, and β' are also variable with λ, but I have proved in the former paper (and it may be accepted immediately as a deduction from Fermat's theorem of the minimum optical path) that the contributions of these to $\frac{\partial W}{\partial \lambda}$ add up to zero, and that they need not therefore be taken into account. Hence IV.* is an alternative equation for the computation of lens-systems in such a way as to attain minimum focus for a prescribed wave-length. It may be modified in a manner similar to that applied to IV., giving

$$\text{VI.}\quad \frac{\partial W}{\partial \lambda} = \frac{\partial n'}{\partial \lambda} y \frac{\sin\frac{\phi'}{2}}{\cos\frac{\alpha}{2}\cos\frac{\beta'}{2}} - \frac{\partial n}{\partial \lambda} y \frac{\sin\frac{\phi}{2}}{\cos\frac{\alpha}{2}\cos\frac{\beta}{2}}$$

an expression which is at least as convenient as that given in the former paper, and which may advantageously be computed as a check.

By computing by the trigonometrical formulæ (1) to (5), and also by V. and VI., we may free a lens-system from spherical and chromatic aberration, although only one single ray has been followed through the system. This seems rather a startling proposition, and might at first sight seem simply absurd; it becomes

acceptable when we consider that the method of optical paths has *one* optical path given without computation, *i.e.* the optical axis itself. This is perhaps one of the most striking illustrations of the power of these new methods of computation.

Further interesting results follow on reducing V. to first-order terms, *i.e.* applying it to rays sufficiently near the optical axis to make the sines equal to their angles, and the cosines equal to unity. Adopting the nomenclature of the former paper, *i.e.* a left-hand index o for these paraxial angles, we obtain easily

$$_{o}W = \tfrac{1}{8}n'\,{}_{o}y\,{}_{o}\phi({}_{o}\beta - {}_{o}\beta')({}_{o}\phi - {}_{o}\beta')$$

It is not difficult to see by referring to fig. 5 that ${}_{o}\beta = \frac{{}_{o}y}{l}$; ${}_{o}\alpha = \frac{{}_{o}y}{r}$; ${}_{o}\beta' = \frac{{}_{o}y}{l'}$ from which by (2) and (4) ${}_{o}\phi$ and ${}_{o}\phi'$ may be found ; introducing these we find

$$\text{VII.}\quad {}_{o}W = \tfrac{1}{8}n'\frac{{}_{o}y^4}{r^2}\cdot\frac{l'-r}{l'}\cdot\frac{l'-l}{ll'}\cdot\frac{ll'-rl'-rl}{ll'}$$

which shows that in the first approximation the differences of phase increase with the fourth power of the aperture. Being algebraical, Equation VII. may possibly supply an alternative analytical solution of the problem of computing a spherically corrected object-glass. The corresponding chromatic equation follows from VI., viz. :

$$\frac{\partial\,{}_{o}W}{\partial\lambda} = \frac{\partial n'}{\partial\lambda}\;\frac{{}_{o}y}{2}\,{}_{o}\phi' - \frac{\partial n}{\partial\lambda}\;\frac{{}_{o}y}{2}\,{}_{o}\phi$$

or, eliminating the angles as above,

$$\text{VIII.}\quad \frac{\partial\,{}_{o}W}{\partial\lambda} = \frac{\partial n'}{\partial\lambda}\;\frac{{}_{o}y^2}{2}\;\frac{l'-r}{l'r} - \frac{\partial n}{\partial\lambda}\;\frac{{}_{o}y^2}{2}\;\frac{l-r}{lr}$$

The differences of phase at focus due to chromatic aberration increase therefore as the square of the aperture.

In conclusion I will use VII. to show the connexion between spherical aberration regarded as a difference of phase and geometrical spherical aberration regarded as a departure of "rays" from the desired direction.

Reference to fig. 1 will make it quite clear that the two stand in the relation of a function and its differential coefficient, *i.e.* that if we have a function expressing the connexion between the ordinates and the distance between ideal and real waves, the differential coefficient of that function expresses the angle between the respective normals, which is a measure of the geometrical aberration.

Before making use of this interesting relation, we must remember that our equations throughout represent *optical* paths ;

in order to get corresponding geometrical distances we must therefore divide them by the index of the medium, which in the case of VII. amounts to leaving out the factor n'. Bearing this in mind and putting the angle between the real and the ideal "ray" as ψ we deduce

$$\psi = \frac{\partial_o W}{\partial_o y} = \tfrac{1}{2}\,{}_o y^3 \cdot \frac{l'-r}{l'} \cdot \frac{l'-l}{ll'} \cdot \frac{ll'-rl'-rl}{ll'}$$

and combining this with VII., we obtain the interesting relation

$$\frac{{}_o W}{\psi} = \frac{n'\,{}_o y}{4} \text{ or IX. } \psi = \frac{4\,{}_o W}{n'\,{}_o y}$$

By this equation we can approximately compute ψ from ${}_o W$, or *vice versa*; as it stands it of course gives ψ in radians and ${}_o W$ in the same unit of length in which y is measured. To obtain ψ in seconds of arc we must multiply the right-hand side with 206,265. If y, the semiaperture, is measured in inches, we may express ${}_o W$ in wave-lengths by dividing the right-hand side by 50,000, the approximate number of average light-waves in one inch; hence our equation for y in inches, ψ in seconds, and ${}_o W$ in approximate wave-lengths becomes, near enough,

$$\psi = 16\,\frac{{}_o W}{n'\,{}_o y}$$

which means that in an object-glass of two inches aperture ($y = 1$) one wave-length of physical aberration corresponds to 16 seconds of geometrical aberration, or a "circle of confusion" of 32 seconds.

Lord Rayleigh has shown that ${}_o W$ may become $\frac{1}{4}$ wave-length without serious deterioration of the image. Accepting this, we deduce for a 1-inch object-glass

$$\psi = \frac{16 \times \frac{1}{4}}{\frac{1}{2}} = 8 \text{ seconds}$$

or a circle of confusion of 16 seconds of arc as the geometrical aberration permissible. As the resolving power of an inch object-glass is about $4\frac{1}{2}$ seconds, this shows most clearly how utterly misleading the geometrical theory is. According to the latter we should have stars spread out into discs nearly four times as large as the resolving power; in reality the image is little worse than that of a theoretically perfect glass.

This is really borne out by practical experience; moderate amounts of spherical aberration are almost past detection at the focus; they only show up when the star image is expanded by "racking in or out." It will nevertheless prove a great surprise to the vast majority of practical opticians who, impressed by the

almost inconceivable smallness of a wave-length, consider it simply absurd to be asked to make the different paths equal within a fraction of this minute quantity; they would, however, be indignant if it were suggested that they could not bring the more familiar geometrical "rays" together within a circle of confusion corresponding in size to the theoretical resolving power of the object-glass. As I have just shown, it is far easier to attain Lord Rayleigh's quarter-wave standard than to reach the circle-of-confusion limit usually adopted in geometrical optics.

Bedford Park, W.:
1905 April 12.

On the General Design of Spectrographs to be attached to Equatorials of large aperture, considered chiefly from the point of view of Tremor-discs. By H. F. Newall.

The following note is an attempt to gain a general idea of the dimensions of that spectrographic installation that forms the turning-point at which increase in size is attended by increasingly disproportionate results. We know how, in the case of object-glasses, an increase in aperture is attended by an increase in absorption; and Vogel's numerical estimates (not to speak of others) have taught us that after the value of approximately nine feet for the diameter of an object-glass is passed, an increase in diameter would be attended by a diminution in brightness. In the case of spectrography we have for some time past recognised the need to use wide slits and high resolving powers. It is of interest to get even a rough estimate of the dimensions, for which, if we in any way design a spectrograph so that its resolving power is proportional to the aperture of the equatorial to which it is attached, the absorption in the spectrograph counterbalances the light-gathering power of the equatorial. The results will help us to decide along what lines of design lies the greatest promise of extension of power.

The paper divides itself roughly into three parts: (i.) a part dealing with the conveniences of estimating slit-widths in terms of visible diffractional phenomena instead of millimetres; (ii.) a part treating of the effects of atmospheric disturbance on star images—in short, of tremor-discs; (iii.) a section in which the attempt is made to reconcile the requirements of the astronomer (in the way of perfection of photographed spectra) with the disadvantageous conditions imposed by atmospheric disturbances.

We collect starlight under conditions that give us, when effects are summed up for a time, circular images or tremor-discs of not inconsiderable dimensions; we measure our photographs

of spectra under conditions that impose the need to use narrow rectangular slits. If the tremor-disc is large compared with the width of the slit only a slice of the circular disc is used, and the total light transmitted is proportional only to the diameter of the object-glass and not to its area ; the economy of light is disregarded. If the slit is widened so as just to include the tremor-disc, or the effective part of it, the light collected is economically used ; but to get the height of spectrum (length of spectral lines) needed for good measurements the star must trail, and the question becomes one of economy of time.

The spectrographic problem has more frequently been dealt with from the point of view of attaching a spectrograph to an existing telescope, and the question of the intensity of the light on the slit has been perhaps unduly emphasised. It has now become a matter of interest to deal with the subject from the point of view rather of transmission of light, and we have to dwell on an equation of continuity. In the following notes I deal chiefly with "transmissions." In our efforts to extend our search for knowledge to faint stars, or to apply very high power to bright stars, we are always struggling with under-exposed photographs ; and, on the ground that the faintest image of a star that is ever attained in astronomical photography is $2''$ in diameter, I have sought to make the spectrographic image of such a star image the standard of comparison, or rather the point of departure ; for such a star image is effectively the smallest that falls on the slit, and we must avail ourselves of as much of the light in it as possible. The balance of those two conflicting elements, the transmission at the slit and the transmission through the prisms, is the really important factor in the design of a stellar spectograph.

In dealing with resolving power and purity I have throughout adhered to the idea originally worked out by Rayleigh and by Schuster, except in so far as I have not adopted Schuster's definition of the *numerical* values, but have simply put $P = \lambda/\delta\lambda$, λ and $\delta\lambda$ being expressed in terms of the same unit of wave-length. This gives, as the unit of purity, that purity which would allow the resolution of lines which differ in wave-length by an amount equal to the mean of the wave-lengths of the two lines. I have refrained from introducing into these simple relations any considerations relating to the range of wave-lengths to be found in the individual bright or dark lines of actual spectra, for these, as it seems to me, relate to the sources of light to be investigated by the spectroscope and do not concern the power of the instrument.

Diffractional Method of Estimating Slit-widths.

Consider diffraction of light through a narrow slit, when a parallel beam (plane wave-fronts) of light falls upon it and the phenomena are studied on a screen placed at a distance, f, from

the slit, s. The distance between the mth dark diffraction band on the right of the centre and the mth dark band on the left is

$$2\xi = \frac{2m\lambda f}{s} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where ξ is the distance from the centre to the mth dark band measured on the screen. In the figure the positions of the 1st, 2nd, 3rd, and mth bands are indicated on both sides of the centre O. If a circle be drawn on the screen with diameter a round O as centre, then the mth dark bands touch the circle when $2\xi = a$.

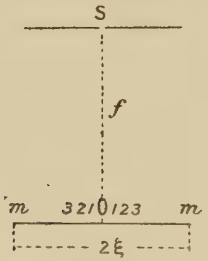


FIG. 1.

And the width of the slit can be adjusted so that this relation holds for any desired value of m. If f, a, λ, and m are known the width of the slit can be estimated in the well-known way, which I shall refer to as the diffractional method of estimating slit-widths. We have

$$s = \frac{2m\lambda f}{a} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

When an object-glass of circular aperture a and focal length f is pointed towards a star, it forms a diffraction image in its focal plane, such that the mth dark diffraction ring has a diameter

$$d = \frac{2p\lambda f}{a} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where p is related to m in the well-known way, as follows :

m = order of ring.	p	m = order of ring.	p
1	1·2197	6	6·2439
2	2·2330	7	7·2448
3	3·2383	8	8·2454
4	4·2411	9	9·2459
5	5·2428	10	10·2463

Hence $2p$ differs from $2m$ by nearly a constant quantity differing little from 0·5 ; and we may write equation (3)

$$d = \frac{(2m+\frac{1}{2})\lambda f}{a} = \frac{2m\lambda f}{a} + \frac{1}{2}\frac{\lambda f}{a} \quad \ldots \quad \ldots \quad (4)$$

Let us make use of the slit and collimator of a spectroscope to study the diffraction phenomena, and let us turn the slit towards a distant source of light ; we shall find the diffraction pattern thrown on the object-glass of the collimator as on the screen and circle of § 1, and we may adjust the slit to any desired width by observing the scale of the diffraction pattern on the object-glass. The method is, I believe, well known to many spectroscopists ; but, as far as I am aware, the convenience of the method is not wholly or generally recognised. Let us suppose that the object-glass has an aperture, a, and focal length, f, as in § 2. Then when the mth band falls on the edges of the object-glass we know that the slit has a width

$$s = \frac{2m\lambda f}{a}$$

and by equation (4) we see that

$$s = d - \frac{1}{2}\frac{\lambda f}{a}$$

The small last term on the right prevents a perfectly concise statement of this relation ; yet the following serves as a good practical rule : When the slit-width is adjusted so that the mth diffraction bands fall on the edges of the object-glass of the collimator, then the slit has a width nearly equal to the diameter of the mth dark ring in the diffraction image of a star as seen with the object-glass of the collimator.

I have adopted the custom of recording slit-widths in terms of these phenomena (*e.g.* slit $m = 1$, or slit $m = 4$) ; and as a name is a convenience I would propose to call m the diffractional indicator.

In stellar spectroscopy it is generally arranged that the ratio A/F of the equatorial used to throw the star's image on the slit is the same as that of the collimator a/f. Under these circumstances the image of the star is a diffraction pattern of the same linear scale as that due to the collimator. Hence we may put the practical rule as follows. When the slit-width is adjusted (in parallel light) so that the mth diffraction bands fall on the edge of the object-glass of the collimator of a star spectroscope, its width is such as to include m maxima in the diffraction pattern of the star's image on the slit. And, moreover, the geometrical image of the slit in the focal plane of the camera includes under these circumstances just as many maxima in the diffraction pattern appropriate to the camera.

The purity of the spectrum depends on the width of the slit used. If P represent purity of the spectrum, R the theoretical resolving power of the spectroscope, and ψ the ratio $a/f = A/F$, we have as the relation connecting purity, resolving power, and wave-length, according to Rayleigh and Schuster

$$P = \frac{\lambda}{s\psi + \lambda} R \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

whence by equation (2) we have

$$P = \frac{1}{2m+1} R$$

If Wadsworth's relation is adopted as being nearer to observed conditions, we have

$$P = \frac{\lambda}{s\psi \cdot \frac{2s\psi - \lambda}{2s\psi + \lambda} + \lambda} R = \frac{1}{2m \cdot \frac{4m-1}{4m+1} + 1} R$$

We have then for various values of m the following numerical relations between P and R :

m.	P (Schuster).	P (Wadsworth).
1	$\frac{1}{3}$ R	$\frac{5}{11}$ R, say $\frac{1}{2}$ R
2	$\frac{1}{5}$ R	$\frac{9}{37}$ R „ $\frac{1}{4}$ R
3	$\frac{1}{7}$ R	$\frac{13}{79}$ R „ $\frac{1}{6}$ R
4	$\frac{1}{9}$ R	$\frac{17}{137}$ R „ $\frac{1}{8}$ R
5	$\frac{1}{11}$ R	$\frac{21}{211}$ R „ $\frac{1}{10}$ R

Thus by mere inspection of the diffraction pattern on the object-glass of the collimator we may estimate what fraction of the theoretical resolving power is being actually used.

Practically the procedure here suggested for adjusting the slit of a spectroscope is as follows. The observer allows sunlight (or light from a distant source of small angular width, such as a tall pinhole gas-flame) to fall direct on the slit of the spectroscope ; the diffraction pattern may be seen on the object-glass of the collimator by the light diffused by dust on the object-glass or by the incomplete polish of it. But a more convenient way is to remove the eyepiece of the spectroscope and to place the eye in the focal plane of the camera in that part of the spectrum which is to be utilised in the observations. If the slit is wide the observer sees a bright patch at the centre of the camera lens, coloured according to the position of his eye in the spectrum ; and when the slit is made narrower the dark diffraction bands are readily seen expanding. It is easy to adjust the slit so that any desired bands fall on the edges of the camera lens or on a diaphragm put over it to cut the aperture down to the same linear value as that of the collimator.

If the spectroscope is attached to an equatorial, it may be inconvenient to dismount it to judge of the width of the slit. In such case it is a good plan to diminish the aperture of the object-glass of the equatorial by means of a diaphragm till its effective diameter as seen from the slit is not greater than that of the Sun. The telescope is then pointed to the Sun and the adjustment of the slit can be readily made as above described.

A table is given on p. 626 showing the relation between the diffractional indicator and the actual slit-width in millimetres for collimators of various angular apertures, viz. $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{19}$, $\frac{1}{20}$.

It is obvious that the readings of the micrometer screw provided for narrowing the slit may be recorded for each slit-width $m = 1$, $m = 2$, &c.; and when once the screw readings are calibrated in this manner (readings being taken only in the direction of *closing* the slit) the observer has complete control of the slit. A great advantage of the method lies in the fact that the dangerous operation of completely closing the slit for the purpose of determining the reading of the screw for zero width is avoided. Here also may be mentioned the need for special attention to the sharpness of the chisel-edge of each jaw of the slit. When the width of a slit is of the order of $0^{mm}{\cdot}02$, the *depth* of the opening (as distinguished from the width and the length) asserts itself;

Fig. 2.

for usually the so-called sharp edge of each jaw has had the inequalities rubbed off, and the sharp edge is reduced to a flat face whose depth may easily amount to one or two hundredths of a millimetre without attracting attention.

The diffractional spreading of the beam that passes through a narrow slit enables us to utilise the whole resolving power of a spectroscope, even when the incident pencil on the slit has an angular aperture quite insufficient to "fill" the collimator and prisms. The admirable performance of a spectroscope when used with narrow slit for looking at sunlight without a condensing lens is well known to spectroscopists; see, for instance, Pulfrich, *Astroph. Jour.* i. p. 346 (*Zeitschr. f. Inst.*, 1894, p. 362): it is due partly to the fact that on account of diffractional spread the whole *width* of the prism system is utilised, and partly to the fact that the whole *height* is not used, and so some of the defects that are due to want of homogeneity or imperfect finish of the surfaces of the prisms and object-glasses do not have the chance of asserting themselves.

These diffraction bands are observed when a parallel beam falls on the slit; in most spectroscopic work it is a convergent beam which falls upon the slit.

With a view of forming some estimate of the amount of light

lost in a stellar spectroscope on account of the diffractional spread of the angular aperture of the beam passing into the slit, and the consequent thrust of much of the light on to the stops in the collimator, I made some experiments in 1896 ; but they seem to be hardly worth putting on record in detail.

It was only quite at the end of a troublesome set of experiments that I found a very elegant way of demonstrating this diffractional spread of the light. It consists in making use of the principle of optical reversibility. Take a collimator and turn the object-glass towards the sky and view it through the slit, the eye being held close to the slit. If the slit is wide the outline of the object-glass is seen equally well defined all round the cell ; but as the slit is narrowed, the edges become blurred and ill defined (on the two *sides* when the slit is held vertical). A further narrowing of the slit results in the apparent spreading out of the circular aperture of the object-glass into an oval figure with well-defined edges at the ends of the shorter axis. Here, again, measurement of the slit shows that diffractional spread begins even when $m > 4$, and is exceedingly marked when $m = 1$. The reader will have no difficulty in working out the details of the analogy if I say that when a slit and collimator are supplied by light from an equatorial the collimator object-glass is taking the place of the eye and looking through the slit at the equatorial object-glass ; and the collimator refuses to transmit any of the light that is spread into the ends of the oval figure of the equatorial lens which it would "see."

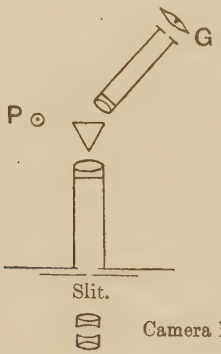


FIG. 3.

The results of the experiment led me to recognise that in stellar spectroscopy of faint stars, for which economy of light is

needed, the slit must have a diffractional indicator not less than $m = 3$, or possibly 4.

To gain further insight into the quantitative loss of light with different slit-widths, the following experiments were made in 1897, with especial reference to the attempt which I contemplated making to photograph the blue line in the spectrum of the corona in the eclipse of 1898. Monochromatic light was thrown upon a slit, and an image of the slit thus illuminated was thrown by means of a camera lens on a photographic plate. For each of a set of various slit-widths a series of these monochromatic images was photographed with different exposures, the plate being moved between successive exposures. An ordinary luminous gas-flame at G was used, and the light was passed through a single-prism spectroscope which was arranged in front of the experimental slit in such a manner that the violet part of the spectrum of the gas-flame fell upon it as indicated in the figure, thus affording constant (and approximately monochromatic) violet illumination of the slit. (In reality it is a narrow strip of continuous spectrum that is used.)

To estimate the width of the slit by the diffractional method a pinhole gas-flame was set up (as at P), so that light from it was reflected from the second surface of the prism on to the experimental slit. A reflecting prism which was placed when required between the camera lens and the photographic plate enabled the observer to adjust the slit-width without disturbing the plate or altering the violet illumination. By inserting an opaque screen near P or near G, either of the operations (*a*) adjusting the slit-width (in approximately parallel light), (*b*) taking photographs (in convergent light) with different exposures, could be completed without interfering with the constancy of illumination in either case ; for neither of the gas flames was touched throughout the whole series of experiments. After a few trials it was easy to arrange a set of exposures which served well for the comparison of the various images.

The resulting photograph exhibited five rows of images with the following exposures (in seconds), the different rows corresponding to the slit-widths indicated by $m = 1$, $m = 2$, &c. :

$m = 1$	20	60	48	36	24	12	60	
$m = 4$	20	15	12	9	6	3	15	
$m = 3$	20	20	16	12	8	4	20	
$m = 2$	20	30	24	18	12	6	30	
$m = 1$	20	60	48	36	24	12	60	60

In comparing the photographic intensities of the various images one comes at the outset upon the well-known peculiarity of the eye, namely, that near the limit of visibility a broad line may be more readily detected than a very narrow one, even when the latter is of stronger intensity. Then with naked-eye

estimates I found representing the image got with 3 seconds exposure and slit-width $m = 4$ by $(m_4, 3^s)$

$$(m_4, 3^s) > (m_3, 4^s) > (m_2, 6^s) > (m_1, 12^s);$$

the images were nearly equally faint, but the widths were disregarded. On the other hand when an eyepiece magnifying ten-fold was used, the widths were obviously different; intensity was then taken as the basis of visibility, and the order indicated above was completely reversed. Accordingly, in estimating the photographic intensities of the images, a microscope with powers 15, 28 was used with the following results:

Table showing Order of Images with respect to Density.

Densest Image.					
20	m_1 60				m_1 60
19	m_1 48				m_1 48
18	m_2 30			m_2 30	
17	m_2 24			m_2 24	
16	m_1 36				m_1 36
15	m_3 20	m_4 20	m_3 20	m_2 20	
14	m_2 18			m_2 18	
13	m_3 16		m_3 16		
12	m_1 24				m_1 24
11	m_4 15	m_4 15			
10	m_4 12				
9	m_3 12	m_4 12	m_3 12		
8	m_2 12			m_2 12	
7	m_4 9	m_4 9			
6	m_3 8		m_3 8		
5	m_1 12				m_1 12
4	m_4 6	m_4 6			
3	m_2 6			m_2 6	
2	m_3 4		m_3 4		
1	m_4 3	m_4 3			
Faintest Image.					

Estimates being then made of step-intervals, the results are displayed in graphical form in fig. 4. The relation between this procedure and what is known as Argelander's method is obvious.

Though there are inconsistencies which are not quite easy to explain (*e.g.* the inclination of the m_2 line and the equality of the 20-sec. exposures for m_2, m_3, and m_4) the results show conclusively that a considerable amount of light is lost when the slit is as narrow as is indicated by $m = 1$. The slit has to be at

least three times as wide before the loss of light ceases to be appreciable. To avoid misapprehension I may reiterate that, though the slit has been adjusted by the diffractional method in parallel light, the photographs were taken with a convergent beam falling on the slit.

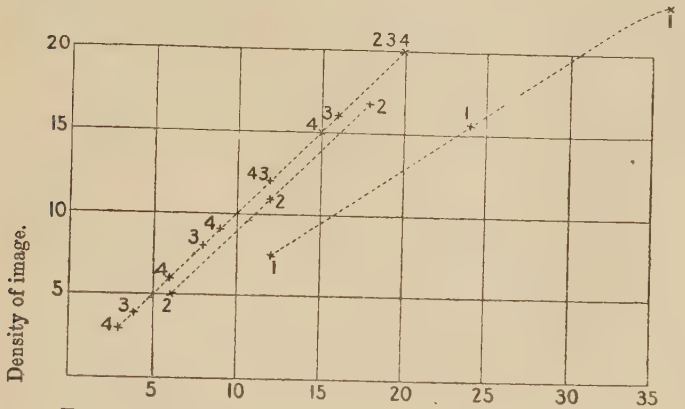


Fig. 4.

Equal photographic densities in the images may be attained with exposures 12 sec. for m_1 and $7^{s}{\cdot}5$ for m_3 and m_4. This indicates that something like $37\frac{1}{2}$ per cent. of the light is lost in the image when the slit-width is $m = 1$. Again, equal densities obtain for exposures 24 sec. for m_1, and 15·5 for m_3 and m_4, indicating a loss of about 35 per cent. The approximate agreement between these results shows that the linear relation assumed between density and intensity is not far wrong for images as dense as those involved. [These results are in satisfactory agreement with those recently obtained by Mr. Moore at the Lick Observatory (*Astroph. Jour.* xx. 285).]

Thus it appears that when a slit-width $m = 3$ is used nearly five times as much light from the tremor-disc of a star passes through the collimator on to the prisms as when a slit-width $m = 1$ is used. For a slit $m = 4$, the figure becomes more nearly six and a half times. More careful measurements are, however, required; for it is probable that the loss by diffraction will depend to some extent upon the angular aperture of the pencil of light incident upon the slit.

As a result of these experiments I have aimed at using slit-widths not less than $m = 3$ whenever economy of light was of importance. The use of so wide a slit in stellar spectroscopy is rendered possible by the fact that the slit is illumined by a tremor-disc of considerable dimensions.

Considerations relating to the Tremor-disc.

Experience shows that it is only under exceptional circumstances that stellar photographs exhibit star-images of smaller diameter than 1″ ; and probably most observers would be content if the smallest well-formed star-images on their photographs did not exceed 3″. This experience has been gained with refractors and reflectors, large and small ; and there is a general belief that the atmospheric tremor is mainly responsible for the fact that smaller images are not usually attainable. We may take it, therefore, that the smallest effective accumulative image of a star on the slit of a star-spectroscope is a "tremor-disc" of diameter 2″ or 3″ at least.

"The name ["tremor-disc"] more or less explains itself ; it is easiest to state what it is intended to convey by reference to a photograph of a star taken with a long exposure. The star image [diffraction pattern] moves about on the plate in consequence of atmospheric tremor, and produces its effect at each spot on which it rests ; the developed image is strongest where the star has most frequently rested. The distribution of density is probably symmetrical about the mean position of the star, and the intensity at different points along a diameter of the resulting tremor-disc is probably fairly well represented by a 'law of errors' curve. Apart from the photograph which shows the summation of effects the tremor-disc may be conceived as existing in time, so to speak ; and the effect produced in a slit spectroscope depends on the relation between a certain area of tremor-disc and the area of the slit illumined by it. The tremor-disc is of greater importance, so far as the design of a stellar spectroscope is concerned, than the diffraction-disc, which has generally been considered" (*Monthly Notices* [1896], vol. lvi. p. 108).

In 1893 I made some attempt to study the way in which a star-image dances about in the focal plane of a telescope. The procedure then was to let a star trail on a photographic plate, and to study the beknottedness of the trail and the deviations in declination. The trails were enlarged photographically about tenfold, and a scale of seconds of arc was ruled on the plate ; or a double star like *Castor* or γ *Virginis* was allowed to trail, in which case the declination difference gave the scale of the deviations. Thus in a trail on a bad night, *e.g.* 1893 September 23, the star frequently deviated from its mean position by 3″ ; in a trail of 27 seconds of time I counted eighty-seven knots, and when a straight line was drawn down the axis of the trail it was seen that forty-six times the star was completely off the axis. The "seeing" during that trail was recorded as "$\frac{1}{5}$ very bad."

On a good night, *e.g.* 1893 September 27, the deviations were often 2″ from the mean position ; the knots were difficult to recognise or count ; but I found the star was thirty-eight times

off the axis in a trail of 26 seconds of time; seeing "very good $\frac{4}{5}$."

When two parallel lines were drawn down the axis of the trail to represent the width of the slit of a spectroscope on the magnified scale I found the following:

		Sec.	On Slit. sec.	Off Slit. sec.
(1) 1893 September 23	Seeing "very bad $\frac{1}{5}$"	Trail 27	11	16
(2) 1893 September 27	„ "very good $\frac{4}{5}$"	„ 26	13	13

In very bad "seeing" the star was frequently crossing the slit; in very good "seeing" the star often remained stationary in declination for a second or more, sometimes on the slit, sometimes off.

These preliminary investigations seemed likely to lead me very far afield, and I even devoted some time to collecting photographs of trails of stars in different parts of the sky on a given night to see whether the direction of the wind made marked peculiarities in the trails, such as a suppression of the knots, which might be attributed to movements in right ascension, or an intensification of the declination movements. But it was clear to me that the important matter was the summation of these movements in time, and so I rested content with the concept of a tremor-disc which ought to guide us in our designs of spectrographs.

Distribution of Light in a Tremor-disc.

We should have perfect "seeing" and a perfect diffraction image of a star if the light from the star fell in plane wave-fronts with uniform distribution of intensity over the whole wave-front. Atmospheric disturbances upset not only the uniformity of distribution of light over an object-glass, but also the planeness of the wave-fronts; each wave-front must be regarded as buckled and corrugated and rippled in a way that changes from moment to moment on any given night, and on a scale that varies from night to night. For the purpose of making a numerical estimate I propose to regard atmospheric disturbances as producing two effects on the distribution of light in the focal plane of a telescope, (1) a scattering of light over a very considerable field (say 40″ or 50″), (2) a movement of the main concentration of light through small distances (say 5″) from the mean position. In the visually observed image we have a dancing star accompanied either by a fairly uniformly illuminated background or by a system of flashing rays; in a photograph we get the time-integral of these changes. For the sake of simplicity it will be convenient to use two names—"tremor-disc" and "scatter-disc."

In the "scatter-disc" we may sum up the light that is to all intents and purposes lost, so far as image is concerned, because it is spread over so wide a field [say 20″ radius (*Monthly Notices*, vol. liv. p. 376)] that its intensity is relatively negligible.

In the "tremor-disc" we sum up the light that forms the image proper; in visual observations we set the wires of a micrometer to bisect an imaginary disc which the dancing momentary image suggests to our conception; the photograph gives us the brighter parts of the tremor-disc as the image of a star.

I shall assume that a definite percentage of the light transmitted by the object-glass is scattered, and that this percentage varies from night to night and is proportional to the diameter of the object-glass (not to its area), and I shall assume that the rest of the light goes into the tremor-disc, and that the dimensions of the tremor-disc are independent of the aperture, and may vary from hour to hour.

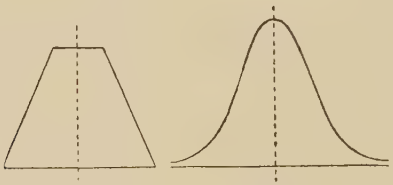


FIG. 5.—Conical. Errors.

For the purpose of numerical estimates I have regarded it as a sufficient approximation to assume, not an "errors-distribution" of light in the tremor-disc, but what I may describe as "truncated conical distribution." The rotation of the figure about the vertical axis gives a surface of revolution whose volume represents the total quantity of light in the tremor-disc. If such a tremor-disc falls on the slit of a spectroscope little more than half the light passes into the slit even when the width of the slit corresponds to twice the "probable error" in the errors-distribution. (I revert below to this analogy.) In the conical distribution it will be obviously desirable to use slits of width not less than the diameter of the core or top of the cone, in any case when the economical use of the light collected by an object-glass is of importance.

The diameter of the base of the cone may be taken as a scant diameter of the tremor-disc: it will be denoted in what follows by τ (measured in seconds of arc); outside of this circle the intensity is zero, but for the scatter-disc. The top of the cone I shall refer to as the "core" of the tremor-disc, and the diameter will be denoted by γ (also in seconds of arc).

Stellar photographs prove that the core is seldom less than $2''$ in diameter. The faintest images begin to appear with this diameter; at first the density of the silver deposit is small, and the effect of increased brightness or increased exposure seems to be only an addition to density without an increase in diameter. The importance of this core γ from the point of view of spectrographic work will be readily grasped.

The following table shows what fraction of the total quantity of light in a tremor-disc of diameter τ'' is collected in the core of diameter γ'' :

TABLE.

τ'' =	2	3	4	5	6	7	8	9	10
$\gamma = 1''$	·43	·23	·14	·10	·07				
$\gamma = 2$		·63	·43	·38	·23	·18	·14	·12	·10

I propose now to deal with two cases : A, that of a slit of width equal to the diameter of the core ; B, that of a slit narrower than the core.

A. *Transmission at a Slit of width equal to the Diameter of the Core of a Tremor-disc.*

Putting I_t for the intensity at the apex of the cone

I_L „ „ core on the level top of truncation

we have $I_t = I_L \times \frac{\tau}{\tau - \gamma}$

The total quantity of light in the tremor-disc is represented by

$$\tfrac{1}{3} I_L \frac{\pi}{4} (\tau^2 + \tau\gamma + \gamma^2)$$

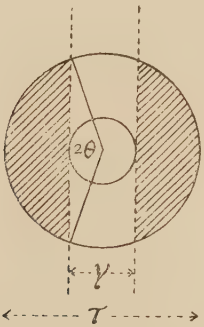


FIG. 6.—Tremor-disc and core on a wide slit.

If the slit of a spectroscope is wide enough to include the whole core of diameter γ we have the quantity of light lost at the slit indicated by the volume of the shaded part of fig. 6 ; treating this as two pyramids each of height I_L and of base $\frac{1}{4}\tau^2 (\theta - \cos\theta \sin\theta)$ $= \frac{\tau^2}{8} (2\theta - \sin 2\theta)$ (an assumption which slightly understates the

loss), we have for the quantity of light lost $\frac{1}{3}I_L \frac{\tau^2}{4}(2\theta - \sin 2\theta)$.

Thus the ratio

$$\frac{\text{quantity lost}}{\text{total}} = \frac{2\theta - \sin 2\theta}{\pi} \cdot \frac{\tau^2}{\tau^2 + \tau\gamma + \gamma^2} \quad \text{where } \cos\theta = \frac{\gamma}{\tau}$$

It will be convenient to call the ratio $\frac{\text{transmitted light}}{\text{total}}$ the "transmission." Numerical values of the transmission are given in the following table for various values of γ and τ. (It must be remembered that the transmission values are slightly overstated in consequence of the approximation used; see, however, below in the section on "Slit narrower than the Core.")

TABLE I.

Transmission at a "Core-wide" Slit.

τ''=diameter of tremor-disc =	2	3	4	5	6	7	8	9	10
Transmission for $\gamma = 1''$	·78	·60	·48	·40	·34	·30	·27	·24	·22
" " 2		90	·78	·68	·60	·54	48	·44	·40

Thus with a tremor-disc $5''$ in diameter a slit transmits only 40 per cent. of the incident light when the core is $1''$ in diameter, and 68 per cent. of the incident light when the core is $2''$ in diameter, if in each case the slit is just wide enough to include the whole core of the tremor-disc ("core-wide" slit).

It is clear that if half the light in the tremor-disc were concentrated in the circular core the radius of the core might, in a sense, be regarded as the analogue of the probable error in an "errors-distribution" of light in the tremor-disc. If, then, we equate the volume of the cylindrical core with that of the coreless frustum, we find

$$\frac{h\pi\gamma^2}{4} = \frac{h}{3}\frac{\pi}{4}(\tau^2 + \tau\gamma + \gamma^2) - \frac{h\pi\gamma^2}{4}$$

whence $\tau^2 + \tau\gamma = 5\gamma^2$

and when $\gamma = 2''$, $\tau = 3''{\cdot}58$

$\gamma = 1''$, $\tau = 1''{\cdot}79$

[It is needless to point out that the long core-wide slit lets through more than the circular core, and thus the transmission 78 per cent. for $\gamma = 2''$ and $\tau = 4''$ recorded in the table is accounted for.]

B. *Transmission at a Slit narrower than the Core.*

Suppose we have a slit whose width ($AA' = 2AO$) is narrower than the diameter of the core. Taking O as origin and OA as axis of x, it will be a sufficient approximation for our

purposes to take the elementary slice of the frustum at A as having a volume

$$I_L \tfrac{1}{2}(2AC + 2AT)dx = I_L(AC + AT)dx$$

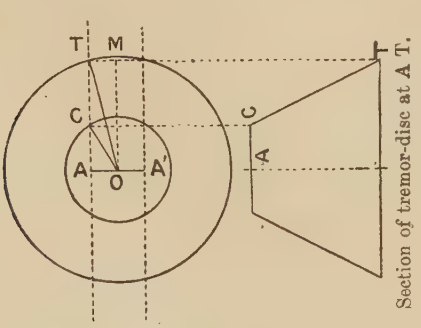


Fig. 7.—Tremor-disc on slit.

OC and OT representing the radii of the core and the tremor-disc. (This value slightly understates the transmission for wide slits.) Now $AC = \left(\frac{\gamma^2}{4} - x^2\right)^{\frac{1}{2}}$ and $AT = \left(\frac{\tau^2}{4} - x^2\right)^{\frac{1}{2}}$

$$\therefore \text{ volume of slice} = I_L\left[\left(\frac{\gamma^2}{4} - x^2\right)^{\frac{1}{2}} + \left(\frac{\tau^2}{4} - x^2\right)^{\frac{1}{2}}\right]dx$$

and the total amount of light transmitted through a slit of width $2x = AA'$ is

$$2I_L \int_0^{AO} \left[\left(\frac{\gamma^2}{4} - x^2\right)^{\frac{1}{2}} + \left(\frac{\tau^2}{4} - x^2\right)^{\frac{1}{2}}\right]dx$$

$$= I_L\left[x\left(\sqrt{\frac{\gamma^2}{4} - x^2} + \sqrt{\frac{\tau^2}{4} - x^2}\right) + \frac{\gamma^2}{4}\sin^{-1}\frac{2x}{\gamma} + \frac{\tau^2}{4}\sin^{-1}\frac{2x}{\tau}\right]_0^{AO}$$

$$\left(\sin^{-1}\frac{2x}{\gamma} = MOC, \text{ and } \sin^{-1}\frac{2x}{\tau} = MOT\right)$$

The whole amount of light in the tremor-disc is

$$\tfrac{1}{3}\frac{\pi}{4} I_L(\tau^2 + \tau\gamma + \gamma^2)$$

The ratio $\frac{\text{transmitted light}}{\text{whole}}$ may be, as before, called the "transmission." Numerical values of the transmission are given in the following table for various values of γ and τ. The values are slightly understated for the wider slits in consequence of the

approximation used; but a comparison with the values given for core-wide slits in Table I. on page 622 shows that the error is probably not more than 3 per cent.

TABLE II.

Transmission at a Slit narrower than the Core.

Slit = γ ×	γ = 1″ ·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	Slit = γ ×	γ = 2″ 1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
2″		·08	·16	·24	·32	·40	·47	·55	·62	·68	·73										
3		·06	·12	·18	·23	·29	·35	·40	·45	·50	·55		·10	·20	·30	·40	·49	·58	·66	·74	·81	·87
4		·05	·09	·13	·18	·22	·27	·31	·35	·39	·43		·08	·16	·24	·32	·40	·47	55	·62	·68	·73
5		·04	·07	·11	·15	·18	·22	·25	·29	·32	·35		·07	·14	·21	·27	·34	·40	·46	·52	·58	·63
6													·06	·12	·18	·23	·29	·35	·40	·45	·50	·55
7													·05	·10	·15	·21	·25	·30	·35	·39	·44	·48
8													·05	·09	·13	·18	·22	·27	·31	·35	·39	·43
9													·04	·08	·12	·16	·20	·24	·28	·32	·36	·39
10													·04	·07	·11	·15	·18	·22	·25	·29	·32	·35

Thus a slit which has a width equal to $\frac{4}{10}$ of the diameter of a core $\gamma = 2''$ transmits 27 per cent. of the light incident upon it if the tremor-disc has a diameter 5″.

[In these estimates of transmission I have included the light that passes into the slit above and below the core, on the ground that though this outlying light in the tremor-disc would not itself produce photographic effect in exposures which are only just sufficient to show the core, yet it contributes to the effect when the core is allowed to trail on the slit, for then the image of the core trails over parts of the photographic plate which have been, so to speak, "prepared" by the previous incidence of the outlying parts of the tremor-disc. The relative feebleness of the outlying parts is well illustrated in an experiment which I made in 1894, and in which a spectrum of *Polaris* was photographed without trail (*Monthly Notices*, vol. liv. p. 524).

In 2 minutes' exposure the width of spectrum was 1″·79

4 " " " " 2·77

10 " " " " 3·42

65 " " " " 3·96.]

Considerations relating to the Proper Apportionment of the Resolving Power of the Spectrograph to the Aperture of an Equatorial for Economic Use of Light in Stellar Spectrography.

It will perhaps give an old question new interest if we deal with it in terms suggested by the diffractional mode of estimating slit-widths described above (p. 610).

We have seen that the slit-width may be expressed by the relation

$$s = \frac{2m\lambda f}{a}$$

and that for minimising what we may call diffractional loss of light in the collimator we must arrange that $m \leqslant 4$.

Now consider a tremor-disc falling on the slit with a diameter of core γ''. For economic use of the light collected by an equatorial the slit must not be much narrower than the core. Let us therefore make the slit "core-wide" and put

$$\frac{F\gamma}{2 \times 10^5} = s = \frac{2m_\gamma \lambda f}{a}$$

where m_γ is the diffractional indicator for a core-wide slit; then, since the ratio A/F for the equatorial is the same as that of the collimator a/f, we reach the result

$$A = \frac{2m_\gamma \lambda \times 2 \times 10^5}{\gamma} \text{ for core-wide slits} \quad \ldots \quad \ldots \quad (6)$$

a relation which gives the aperture of the equatorial suitable for economic and efficient work in terms of the wave-length of light and m and γ. [Thus for spectra to be measured in the region of H_γ, we have, putting $\lambda = 434 \times 10^{-6}\,mm$, and $m = 3$,

$$A = \frac{520}{\gamma}\,mm$$

or, say, twenty inches for $1''$ core of tremor-disc, and 10 inches for $2''$ core of tremor-disc.]

If we wish to use large apertures economically we must adjust m to suit γ. I have summarised matters in Table III.

TABLE III.

$$A = \frac{17{\cdot}36\, m_\gamma}{\gamma} \text{ for } H_\gamma \text{ and core-wide slit. (A in cms.)}$$

A		m_γ for $\gamma = 1''$.	m_γ for $\gamma = 2''$.	A		m_γ for $\gamma = 1''$.	m_γ for $\gamma = 2''$.
Cms.	Inches.			Cms.	Inches.		
30	11·8	1·7	3·4	91	35·9	5·2	10·4
63	24·8	3·6	7·2	102	40·2	5·9	11·8
80	31·5	4·6	9·2	150	59·1	8·6	17·2

A small table is added for convenience in converting the diffractional indicator into millimetres for H_γ and for equatorials of various angular apertures.

m =	1	2	3	4	5
Ratio A/F					
	mm	mm	mm	mm	mm
$\frac{1}{10}$	0·009	0·017	0·026	0·035	0·043
$\frac{1}{15}$	·013	·026	·039	·052	·065
$\frac{1}{15}$	·017	·033	·050	·066	·083
$\frac{1}{20}$	·017	·035	·052	·070	·087

It is at once obvious that for the economic utilisation of light wide slits are necessary, and consequently for a given purity of spectrum high values of resolving power are needed in the spectroscope when equatorials of large aperture are used. *It will be observed that the length of collimator under the condition* $\frac{a}{f} = \text{const.}$ *does not affect the value of* m_γ. It will be a convenience in considering the question to introduce the term "collimator intensity" to express the quantity of light per square centimetre on the object-glass of the collimator. We may then regard the equatorial and collimator as a system for producing an intense beam of star light to be dealt with by an objective prism camera. It must, however, be remembered that the result $A = \frac{17{\cdot}36m_\gamma}{\gamma}$ shows that the use of a large aperture A carries with it the disadvantage of a correspondingly large value of m_γ and the necessity for a large resolving power in order that the required purity of spectrum may be attained.

I shall not now consider the exact relationship between purity and slit-width, but shall assume that the relation given above, viz.

$$P = R/(2m + 1)$$

s one that represents the facts closely enough for my present purpose, even when the slit-width is as large as that implied by diffractional indicators as large as $m = 10$. This, however, is a point which requires careful investigation.

[We may put the point in the following way. If the purity in the spectrum at λ is P, then the centres of resolved images must be λ/P tenth-metres apart in the spectrum. Taking into account the fact that in the diffractional image of a slit of finite width the intensity on the edge of the geometrical image is reduced to one half, we may take it that if the geometrical images are less than λ/P in width, there will be resolution of some kind. Now the actual width of the geometrical image of the slit is $\frac{F\gamma}{2.10^5} \cdot \frac{f_{cam}}{f_{coll}}$ $= \frac{A\gamma}{2.10^5.\beta}$. The value of this in tenth-metres is $\frac{A\gamma}{2.10^5.\beta} \div \frac{ds}{d\lambda}$,

where $ds/d\lambda$ is the linear dispersion. We shall see below that $\beta = (2m+1)P\Big/\frac{ds}{d\lambda}$, and for core-wide slits $\beta = (2m_\gamma+1)P\Big/\frac{ds}{d\lambda}$.

Hence, for resolution of some kind, we must have

$$\frac{A\gamma}{2 \cdot 10^5(2m_\gamma+1)} < \lambda$$

Now we have seen above in equation (6) that $\frac{A\gamma}{2 \cdot 10^5 \cdot 2m_\gamma} = \lambda$. Hence the point we have to consider may be put in this form: Is one part in $2m_\gamma$ parts enough to ensure resolution? I shall assume for the moment that it is; for though Wadsworth's observations on resolution with wide slits should be enough for the settlement of the question, his results lead to the idea that with slits as narrow as about $m = 1$ there should be a maximum in the resolution, as the slit is made narrower, and I have not been able to verify the existence of such a maximum in spite of careful search for it. I do, however, find that when R is as large as 78,000 the purity is *at least* as good as that which would be expected from the relation $P = R/(2m+1)$ for values up to $m = 3$ and $m = 4$.]

Thus far we have only considered the economical use of the light collected by the object-glass and to be transmitted through the slit, and we have seen how atmospheric disturbances impose upon us the need to use a definite width of slit in stellar spectrography if economy of light is of importance. Let us now proceed to state what are our minimum requirements in the perfection of a photographed spectrum, and then see how we can satisfy them with due regard to economical use of light.

Requirements in Photographed Spectra.—It seems to me not amiss to start with the following statement of requirements in a photographed stellar spectrum:

(1) For the proper identification of lines, a purity of spectrum $P = \lambda/\delta\lambda$ of at least 10,000 is needed, allowing of distinction between lines for which $\delta\lambda = 0\lambda{\cdot}4$ at $\lambda 4000$, and $0\lambda{\cdot}5$ at $\lambda 5000$. Thus

$$P = 10{,}000$$

(2) For the proper measurement of wave-lengths, a linear dispersion of at least 1 mm. per 10 tenth-metres is needed. Thus

$$\frac{ds}{d\lambda} = 10^6$$

(3) For the proper discrimination between real stellar lines and faults due to defects in emulsion, a height of spectrum (= length of lines) of at least $0^{mm}{\cdot}25$ is needed.

$$h' = 0^{mm}{\cdot}25$$

With respect to (1) we have seen (p. 612) that the purity of the spectrum is given by the relation $P = R/(2m+1)$, m being the diffractional indicator for the slit used, and R the resolving power of the spectrograph. If we are to utilise the greater part of the light in a tremor-disc, we must have $P = R/2m, +1$. Thus Table III. tells us that for a 25-inch object-glass R must be 154,000 to give P = 10,000 with a core-wide slit.

With respect to (2), viz. the linear dispersion, bearing in mind that $R = a d\theta/d\lambda$, where a is the linear aperture of the collimator, and $d\theta/d\lambda$ is the angular dispersion produced by the prism-system, we see that the linear dispersion is

$$\frac{ds}{d\lambda} = f_{\text{cam}} \cdot \frac{d\theta}{d\lambda} = f_{\text{cam}} \cdot \frac{R}{a} = \frac{R}{\beta} = \frac{(2m+1)P}{\beta}$$

and the angular aperture of camera is

$$\beta = (2m+1)P \Big/ \frac{ds}{d\lambda} \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

We must arrange that β has a practicable magnitude.

With respect to (3), viz. the height of spectrum and the duration of trail of the star on the slit needed to give the required height h', we can deal with the question for the moment in terms of an undispersed image of the slit on the photographic plate in the camera. If the slit is illuminated by a tremor-disc whose core is γ'' in diameter, then the height of the slit illuminated is $\frac{F\gamma}{2 \cdot 10^5}$, where F is the focal length of the equatorial; and the height of the image of the slit as photographed in the camera is

$$\frac{F\gamma}{2 \cdot 10^5} \cdot \frac{f_{\text{cam}}}{f_{\text{coll}}} = \frac{A\gamma \cdot f_{\text{cam}}}{2 \cdot 10^5 \cdot a} = \frac{A\gamma}{2 \cdot 10^5 \cdot \beta} \quad \ldots \quad (8)$$

Hence if the duration of trail is made to be $h' \Big/ \frac{A\gamma}{2 \cdot 10^5 \cdot \beta}$ times as long as the time required for a stationary image, we shall get an image of slit of length h' and with intensity nearly the same as that of the stationary image. Supposing that the photographic density is proportional to It, if an image of intensity I rests on the plate for the time t, then in a given time we shall have the intensity with trail to give $h' = \frac{A\gamma}{2 \cdot 10^5 \cdot \beta \cdot h'} \times$ the intensity without trail. [The intensity with trail will be somewhat greater, inasmuch as we may suppose the exposure for the stationary image to be just long enough for the core alone to impress the plate, whereas in the trailed image the outlying parts of the tremor-disc above and below the core contribute something

to the final density by "preparing" the plate for the trailing core.]

Since the performance of different spectrographic installations is to be gauged under the same conditions of constancy of P, $ds/d\lambda$, and h' for every installation, and since these conditions imply that the area of the resulting photographed spectrum, viz. $h' \times (\lambda_1 - \lambda_2) ds/d\lambda$, is to be the same in every case, the efficiencies of the installations in the given conditions may be compared by comparison of the intensities attained by a given exposure. We may therefore for the moment leave dispersion out of account provided we allow for any alteration that may be brought about by the use of different prism-systems in the transmission of light. We may denote by $\rho\Pi$ the transmission of the prism-system, where ρ is the factor relating to the light that escapes reflexion at the prism-surfaces, and Π is the factor relating to the light that escapes absorption inside the prisms; and for the moment we will assume that Π sufficiently nearly represents the transmission if it has the value corresponding to a plate of glass of thickness equal to half the total base of the prism-system (I revert to this later).

Intensity of Photographed Spectrum with "Core-wide" Slit (P const., $\frac{ds}{d\lambda}$ const., h' const.).—If light of unit intensity from a star falls upon the object-glass of an equatorial of aperture A, the quantity of light transmitted is $\pi A^2 O$, where O is a coefficient of transmission and may be taken from Vogel's table (*Astroph. Jour.* v. 89), modified if need be to take account of the light diverted into the "scatter-disc." The light is collected in a tremor-disc which falls on the slit of the spectroscope. The slit is adjusted so as to allow the whole core of the tremor-disc to pass. The slit-transmission S_γ under these circumstances may be taken from Table II. above (page 624).

The collimator of aperture a (under the usual condition $\frac{A}{F} = \frac{a}{f_{coll}}$) transmits $\pi A^2 O S_\gamma$, and $\frac{\pi A^2 O S_\gamma}{\pi a^2} = \frac{A^2 O S_\gamma}{a^2}$ is the "collimator intensity," it being understood that the slit has a diffractional indicator m_γ appropriate to A (Table III., page 625).

The collimator intensity is $\rho\Pi$-fold by the passage through the prism-system; and thus the intensity of the light incident on the object-glass of the camera is

$$\frac{A^2 O S_\gamma \rho \Pi}{a^2} = K$$

In an exposure t the quantity of light $Kt\frac{\pi}{4}a^2$ passes into the camera. If the whole of it were concentrated in the geometrical image of an element of the core-wide slit, of width $\frac{A\gamma}{2 \cdot 10^5 \cdot \beta}$

and of height $\frac{A\gamma}{2 \cdot 10^5 \cdot \beta}$, the average intensity in the image would be

$$K t \frac{\pi}{4} a^2 \cdot \frac{4 \cdot 10^{10} \cdot \beta^2}{A^2 \gamma^2}$$

But we wish to produce a slit-image of which the height is h' on the photograph ; therefore we must let the tremor-disc trail on the slit for a time, which is $\frac{h'\beta \cdot 2 \cdot 10^5}{A\gamma}$ times as long as the exposure needed to give a stationary image of the core with the required density. Or if we give the same exposure in both cases, the intensity with trail is reduced $\frac{h'\beta \cdot 2 \cdot 10^5}{A\gamma}$-fold. [The circular image of the core is here allowed to trail along the length of the core-wide slit ; the result will be that the intensity is greatest along the central line of the slit, and falls off on either side of it ; and this uneven though symmetrical distribution will be emphasised rather than diminished by the light in the outlying parts of the tremor-disc. We shall have an increase in the effective purity in the spectrum, but this is a refinement beyond my present purpose.]

Hence, for a spectrum of height h' to be obtained in a given time, the intensity will be

$$A^2 O S_\gamma \cdot \rho\Pi \cdot \frac{\pi}{4} \frac{\beta \cdot 2 \cdot 10^5}{A\gamma \cdot h'}$$

Remembering equation (7) page 628, viz. $\beta = (2m_\gamma + 1)P / \frac{ds}{d\lambda}$

and also equation (6), page 625, viz. $2m_\gamma\lambda = \frac{A\gamma}{2 \cdot 10^5}$

we get for core-wide slits (h' const. in a given exposure)

$$\text{the intensity} = A^2 O S_\gamma \cdot \rho\Pi \cdot \frac{\pi}{4} \frac{(2m_\gamma + 1)P}{h' \cdot \frac{ds}{d\lambda} \cdot 2m_\gamma\lambda} \quad \ldots \quad (9)$$

For even a 25-inch object-glass $2m_\gamma = 14{\cdot}4$; hence $\frac{2m_\gamma + 1}{2m_\gamma}$ differs little (7 per cent.) from unity, and approaches more nearly to unity the larger the aperture of the equatorial. Therefore for a comparison of large installations we have to deal with the value of $A^2 O S_\gamma \cdot \rho\Pi$, the other factors being arbitrary constants. Now with regard to the prism-transmission, which is represented by $\rho\Pi$, we must bear in mind that

$$R = (t_2 - t_1) \frac{d\mu}{d\lambda} = 2Na \frac{\tan i}{\mu} \frac{d\mu}{d\lambda}$$

and hence $\frac{1}{2}(t_2 - t_1)$, or the mean thickness of glass traversed in

the prism-system, is equal to $Na\frac{\tan i}{\mu}$, if N prisms are used to transmit a collimated beam of diameter a; and if the prisms are all of such angle that the reflected light is wholly polarised, then $\tan i = \mu$, and $\frac{1}{2}(t_2 - t_1) = Na$. Under these circumstances ρ approaches to the value $\frac{1}{2}$ as N increases (for 3 prisms it is about ·65 for $\mu = 1{\cdot}6$); and $\Pi = a^{Na}$, where a is the coefficient of transmission for the material used for the prisms. Again $R = (2m_\gamma + 1)P$, which differs but slightly from $2m_\gamma P$ or $\frac{A\gamma}{2 \cdot 10^5 \lambda} \cdot P$, when $2m_\gamma$ is large. Therefore for polarising prisms ($\tan i = \mu$) we have

$$R = 2Na\frac{d\mu}{d\lambda} = \frac{A\gamma}{2 \cdot 10^5 \lambda} P$$

and we may put $Na = kA$.

Hence we may write

$$A^2OS_\gamma \rho \Pi = A^2OS_\gamma \rho a^{kA}$$

Before discussing this with a view to finding the value of A which gives the maximum value of intensity on the photograph, it will be well to consider the case of slits narrower than the core of the tremor-disc. I would only point out, with regard to an increase of power of the spectrograph, that if with $Na = kA$ we keep N constant (as is desirable when we have chosen N so as to give the convenient deviation of the beam) then we get the desired increase in Na by increase of a, and a corresponding increase in the size of the prisms.

Intensity of a Photographed Spectrum with Slit narrower than the Core (P const., $\frac{ds}{d\lambda}$ const., h' const.).—Suppose the slit allows only part of the core to pass, say $\sigma = n\gamma$, n being less than unity; then the linear width of the slit is

$$s_{n\gamma} = \frac{F \cdot n\gamma}{2 \cdot 10^5} = \frac{2nm_\gamma F}{A} = \frac{2m_{n\gamma} F}{A}$$

and it is clear that $m_{n\gamma} = nm_\gamma$.

The collimator intensity is now $\frac{A^2OS_{n\gamma}}{a^2}$ with a diffractional indicator nm_γ.

The intensity on the camera lens is now $\frac{A^2OS_{n\gamma} \cdot \rho'\Pi'}{a^2} = K'$.

The geometrical image of an element of the slit has now a height $\frac{A\gamma}{2 \cdot 10^5 \cdot \beta'}$ and a width $\frac{An\gamma}{2 \cdot 10^5 \cdot \beta'}$. If we wish the height of the spectrum to be h' as before the star must trail for a time $\frac{h'\beta' \cdot 2 \cdot 10^5}{\gamma A}$.

We now have for the camera

$$\beta' = (2nm_\gamma + 1)\mathrm{P} / \frac{ds}{d\lambda}$$

Hence for slits narrower than the core (h' const. in a given exposure), the intensity is

$$\mathrm{A}^2\mathrm{OS}_{n\gamma} \cdot \rho'\Pi' \cdot \frac{\pi}{4} \frac{(2nm_\gamma + 1)\mathrm{P}}{h' \cdot \frac{ds}{d\lambda} \cdot 2nm_\gamma\lambda} \quad \ldots \quad \ldots \quad 10)$$

It is clear that this expression gives, when $n = 1$, the value of the intensity for a core-wide slit, and it agrees as it should with the expression (9, page 630) previously got.

We may therefore take as the expression for the intensity of a photographed spectrum under the specified conditions and assumptions

$$\mathrm{A}^2\mathrm{OS}_{n\gamma} \cdot \rho\alpha^{\frac{(2nm_\gamma+1)\mathrm{P}}{2\frac{d\mu}{d\lambda}}} \cdot \frac{\pi}{4} \cdot \frac{(2nm_\gamma + 1)\mathrm{P}}{h' \frac{ds}{d\lambda} 2nm_\gamma\lambda}$$

Assuming that the common atmospheric conditions at Cambridge are such as to imply a tremor-disc 5″ in diameter, and that the experience of those who have carefully photographed star-images is such as to imply that the tremor-disc has an effective core seldom or never less than 2″ in diameter, I have calculated (i.) the value of m_γ for a core 2″ diameter for various apertures $\left[m_\gamma = \frac{\mathrm{A}\gamma}{17 \cdot 36} \text{ for } \mathrm{H}_\gamma\right]$, (ii.) the resolving power needed to give a purity of 10^4 $[\mathrm{R} = (2m_\gamma + 1)\mathrm{P}]$, (iii.) the absorption along the axis of prisms, using Vogel's value for the transmission of Jena glass O. 102 ; and assuming that the required resolving power can be got by adjusting properly the values of a and $d\theta/d\lambda$, I have treated ρ as constant, and have prepared the following tables.

Table IV. shows the relative intensities gained in the photographed spectra with installations of different apertures, when the spectrograph is designed in each case to give purity 10^4 $ds/d\lambda = 10^6$ $h' = \frac{1}{4}$ mm. with a slit adjusted in each case to include the whole core ($\gamma = 2''$) of the tremor-disc ($\tau = 5''$). No allowance is made in this table for diffractional losses in the collimator, for in no case does the diffractional indicator fall below 3. In giving the numerical values of the photographic intensity I have omitted the factor $\rho \cdot \frac{\pi}{4} \cdot \frac{\mathrm{P}}{h' \frac{ds}{d\lambda}\lambda}$ as common to all.

TABLE IV.

Relative Intensities of Spectra given by Various Installations.

(Spectrum near H_γ. Purity = 104, $\frac{ds}{d\lambda} = 10^6$, $h' = \frac{1}{4}$ mm. Prisms of Jena glass O. 102. Core-wide slit. Tremor-disc $\tau = 5''$, core $\gamma = 2''$.)

Aperture. Cm. (Inches.)		$m\gamma$.	A^2.	O.	Sy.	$\Pi(O\ ^{\cdot}102)$.	$\frac{2m\gamma+1}{2m\gamma}$.	Photographic Intensity.	β.	R.
30	(12)	3·4	900	·64	·63	·450	1·15	189	·078	78000
50		5·8	2500	·57	·63	·278	1·09	271	·126	126000
63	(25)	7·2	3969	·53	·63	·208	1·07	294	·154	154000
70		8·1	4900	·50	·63	·170	1·06	278	·172	172000
80	($31\frac{1}{2}$)	9·2	6400	·47	·63	·138	1·05	275	·194	194000
91	(36)	10·4	8281	·44	·63	·108	1·05	260	·218	218000
102	(40)	11·8	10404	·40	·63	·086	1·04	234	·246	246000

In calculating O from Vogel's table (*Astroph. Jour.* v. 89) I have allowed for a diversion of a certain percentage of the light out of the tremor-disc into the scatter disc, on the scale of 1 per cent. for every 10 cm. in the aperture. In the last columns of the table I have entered β and R. Under the assumed conditions of purity and linear dispersion, we have $\beta = \frac{2m_\gamma + 1}{100}$, and $R = \beta \times 10^6$.

We thus see how destructive is the effect of absorption in the prism-systems when we try to utilise 63 per cent. of the light thrown by the equatorial on the slit. We have already reached the turning-point of advantage to be gained from large aperture under these conditions when we have arrived at a 25-inch object-glass. We must narrow the slit and put up with smaller slit-transmission in order to attain greater prism-transmission, or else lower our standards in the photographs.

Table V. shows the relative intensities for different installations with the slit adjusted to include various fractions ($\cdot1$, $\cdot2$, . . . 1·0) of the core ($\gamma = 2''$) of the tremor-disc ($\tau = 5''$), when the spectrograph in each individual case is designed to give purity = 10^4, $ds/d\lambda = 10^6$, $h' = \frac{1}{4}$ mm. No allowance has been made in this table for diffractional losses in the collimator ; the reader may allow for it in cases where nm_γ falls below 3, on a scale not differing much from the following, which I have based on Moore's observations (*Astroph. Jour.* xx. 289).

for $nm_\gamma = 1$, the diffractional loss is 30 per cent.

" " = 2 " " " " 20 " "

" " = 3 " " " " 13 " "

" " = 4 " " " " 7 " "

TABLE V.

Relative Intensities of Spectra given by Various Installations.

(Spectrum near H_γ. Purity = 10⁴, $ds/d\lambda = 10^6$, $h' = \frac{1}{4}$ mm. Prisms of polarising angle and of Jena glass O. 102. Tremor-disc $\tau = 5''$, core $\gamma = 2''$.)

Aperture. Cm.	(Inches.)	m_γ.	$n = \cdot1$. $S_{n\gamma} = \cdot07$.	$\cdot2$. $\cdot14$.	$\cdot3$. $\cdot21$.	$\cdot4$. $\cdot27$.	$\cdot5$. $\cdot34$.	$\cdot6$. $\cdot40$.	$\cdot7$. $\cdot46$.	$\cdot8$. $\cdot52$.	$\cdot9$. $\cdot58$.	$1\cdot0$. $\cdot63$.
30	(12)	3·4	83	108	134	146	161 B	171	177	186	190 3·1	189
50		5·8										271
63	(25)	7·2	197	266	315	332 C	352 3·6	348	340	335	315	294
70		8·1										278
80	(31½)	9·2	245	335 P	380	388	401 4·6	387	367	335	305	274
91	(36)	10·4										260
102	(40)	11·8	308	401	438 3·5	426 Y	419	376	333	271	263	234

In the above table n represents the fraction of the core included by the slit, $S_{n\gamma}$ the corresponding slit-transmission, the figures in a horizontal line represent the intensities for the aperture mentioned in the first column. Under the maximum value is given the diffractional indicator, and under another value a letter is entered to show that an installation exists of dimensions and resolving power suited to work at that value. Thus B indicates Bonn, where Professor Küstner has lately installed an instrument of resolving power about 43,000; allowance should be made for diffractional loss in the collimator for all the values of n, except the two highest, for this installation; and similarly in other cases whenever $nm_\gamma < 3$. C indicates my own installation at Cambridge; P = the new installation at Potsdam, Y = Yerkes Observatory. At Potsdam, Yerkes, and Bonn the prisms are actually made of Jena glass O. 102; at Cambridge they are made of lighter and, I believe, more transparent glass (see page 639).

To illustrate the use of the table, take the case of Cambridge (63 cm.). A photographic intensity of 352 might be attained with prisms of Jena glass O. 102, if the resolving power were $R = (2nm_\gamma + 1)\,P = (2 \times \cdot5 \times 7\cdot2 + 1)10^4 = 82000$, and the diffractional indicator of the slit width were $\cdot5 \times m_\gamma = 3\cdot6$.

[I am actually using R = 77000, and $m = 3$, and, as I believe, a more transparent glass than Jena O. 102.]

If I could utilise a bright third-order spectrum of a 14438 Rowland grating with ruled surface 3 inches by 2 inches, I should have R = 130000; and thus for P = 10⁴, $(2nm_\gamma + 1) = 13\cdot0$, whence $n = \cdot83$, since $m_\gamma = 7\cdot2$.

In this case I should have $S_n = 54$ per cent. instead of

34 per cent.; and the "grating transmission" G would have to be

$$G = \frac{S_{\cdot 5y} \times \rho\Pi}{S_{\cdot 8y}} = \frac{34 \times \frac{1}{2} \times \cdot 431}{54} = \cdot 136 \text{ or, say, } 14 \text{ per cent.}$$

in order to give me a spectrum as bright as that indicated by 352, the maximum attainable by the use of prisms of Jena glass O. 102.

Fig. 8 shows the results of Table V. plotted in graphical form. The dotted lines are drawn through points where the

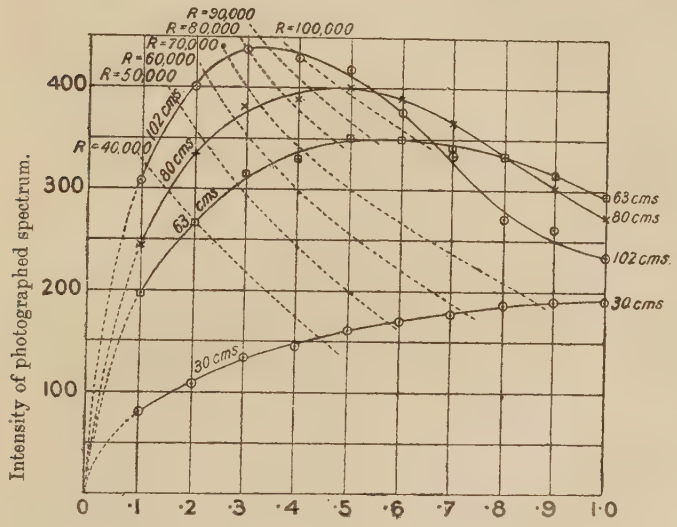


Fig. 8.

resolving power (total length of prism base) has the values marked at the top of the lines.

I recognise very fully that in many ways the discussion just given is far from complete. I embarked upon it, partly, in order to show how helpful the diffractional indicator, which probably every spectroscopist has often seen in the way described above, may be in giving one a general view over the subject; and partly in order to attempt to investigate how close we already are to the limits of what can be achieved with prismatic spectrographs attached to large refractors.

I am convinced that a 30-inch reflector, properly combined with a diffraction-grating spectrograph, would give results which would compare favourably with the most powerful existing installations.

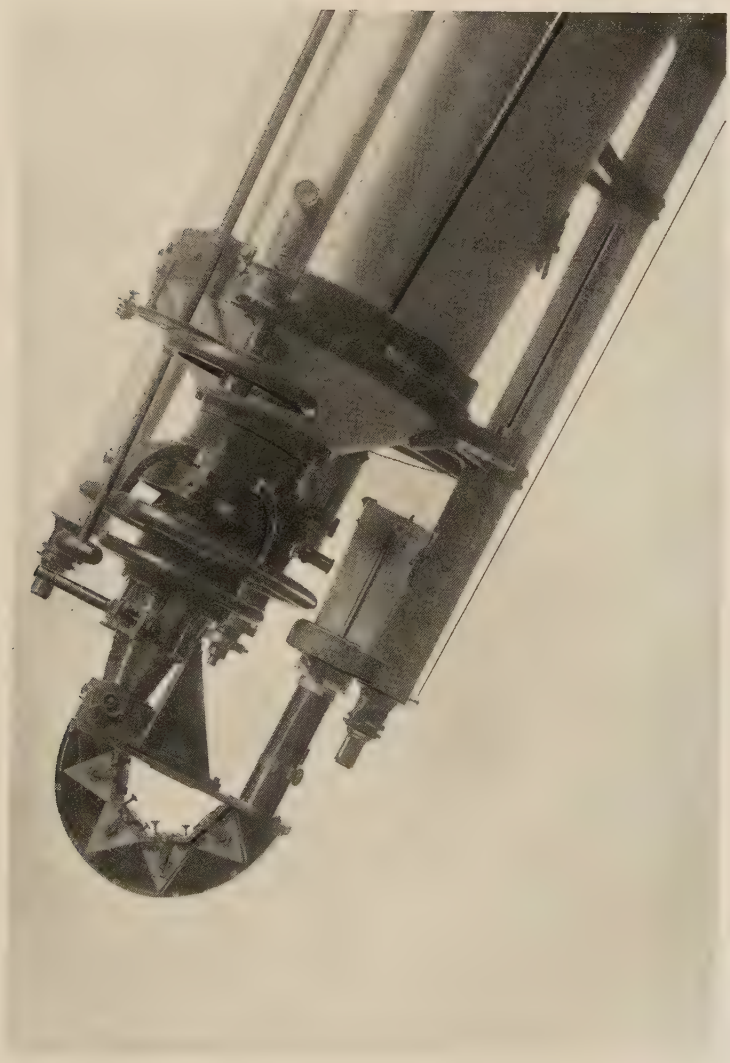
It is almost needless for me to add that in refraining from any attempt to compare different installations under different conditions of seeing I have neglected what is probably a very important consideration.

Description of a Four-prism Spectrograph attached to the 25-inch visual Refractor (the Newall Telescope) of the Cambridge Observatory. By H. F. Newall.

Since 1899 July the spectroscopic work at the Cambridge Observatory has been done with a four-prism spectrograph attached to the 25-inch equatorial (the Newall telescope). A variety of circumstances has stood in the way of the publication of any description of the instrument. The present note is intended to make this good, but it should be stated that the instrument described is avowedly of an experimental form which has persisted longer than was intended. Inasmuch, however, as a considerable amount of work has been done with it, and is being discussed with a view to publication, it seems desirable to publish a description.

The Bruce spectroscope, a single-prism instrument, was constructed in 1895 for use in connexion with the 25-inch equatorial (*Monthly Notices*, vol. lvi. p. 98, and *Astroph. Jour.* vol. iii. p. 266). This form was adopted in the hope that it would be possible with it to obtain photographs of spectra of stars of 4th and higher magnitudes, and it was found that stars of the 4th magnitude were just accessible, with exposures of a reasonable duration, with the narrow slit, which it was desirable to use for the sake of getting adequate purity of spectrum. I hoped to complete observations for velocity of 100 stars, but the instrument was dismounted in 1899 July after getting material for determining the velocity of eighty-five stars.

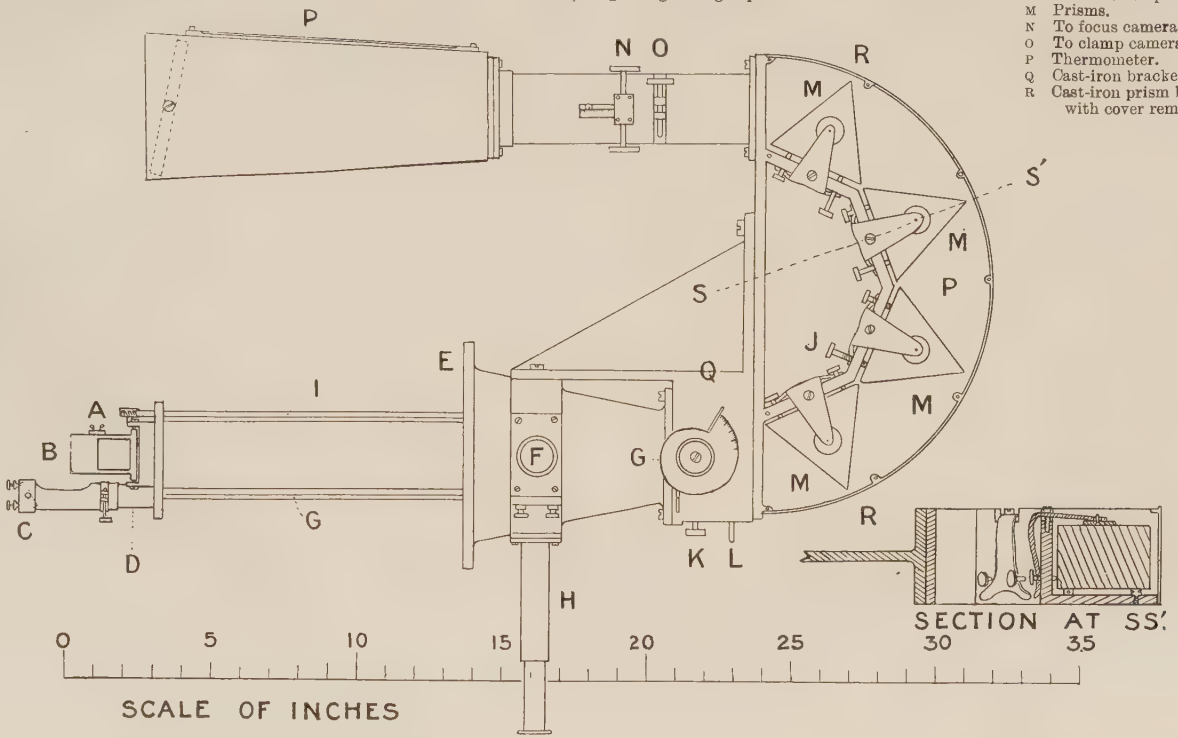
In recording a few of the results at first obtained with the single-prism instrument (*Monthly Notices*, vol. lvii. p. 567) I pointed out that one of the chief difficulties encountered lay in the true identification of lines in the more or less complex groups of lines chosen for bisection in the micrometer; and on p. 575, *loc. cit.*, I stated: "One point is made clear by the investigation—viz. that little or no advantage, so far as the elimination of the error of identification is concerned, would be gained by *doubling* the dispersion and resolving power of the spectroscope, for the disturbing lines in the groups detected would not be separated from the iron lines without a considerably greater increase." It was determined, therefore, to prepare for attaching a powerful train of four prisms, and meanwhile to continue the work with the Bruce spectrograph. Accordingly, when a share in the work of the English expeditions for the Indian Eclipse of 1898 January 22 was allotted to me, the design of the spectroscope to be used for photographing the spectrum of the corona was completed in such a manner that the train of prisms could also be attached to the frame of the Bruce spectrograph, and could thus be used for stellar spectrography. This plan was carried out, and the resulting instrument is now to be described. The great resolving power available when four large prisms are



A FOUR-PRISM SPECTROGRAPH ATTACHED TO THE NEWALL TELESCOPE

Description of Plate

A Red lamp to illuminate slit and guiding comb.
B Comparison prisms.
C Mirror to reflect light into guiding eyepiece.
D Plane through slit.
E Flange by which spectroscope is attached to equatorial.
F Tube to which are fitted the apparatus for comparison spectra.
G To move guiding comb in front of slit.
H To view slit, for guiding during exposure.
I To widen slit.
J To adjust prism.
K To focus collimator.
L To move comparison prisms.
M Prisms.
N To focus camera.
O To clamp camera.
P Thermometer.
Q Cast-iron bracket.
R Cast-iron prism box, with cover removed.



used makes it possible with faint stars to widen the slit; the serious losses which arise in the collimator by reason of the diffractional spread of the beam are thus in great measure avoided. One can overcome the difficulties of identification of lines by utilising the knowledge gained in the study of the spectra of bright stars with both wide and narrow slits.

Plates 18 and 19 represent the four-prism spectrograph, and by comparison with the figure of the Bruce spectrograph (*Monthly Notices*, vol. lvi. plate 10, p. 100) it will be seen that the mode of attachment to the equatorial is the same as for the single-prism instrument; the change consists in the substitution of a train of four prisms held in a strong cast-iron box, R, of semicircular outline in ground plan. This prism box is attached by a strong cast-iron bracket, Q, to the heavy conical casting of gun-metal referred to in *Monthly Notices*, vol. lvi. p. 102, as the framework of the spectroscope. The work has been carried out by the Cambridge Scientific Instrument Company.

The Equatorial and the Correcting Lens.

The 25-inch (63 cm.) equatorial has a focal length of 29 feet (8·84 m.). A simple convexo-concave lens of dense flint-glass, of aperture 5 inches (12·7 cm.) and of focal length 154 inches (391·0 cm.) for light of wave-length 5890–6, is inserted in the convergent beam of rays coming from the object-glass of the equatorial at a distance of about 62 inches (157·5 cm.) from the focus. The corrected focus is nearer to the object-glass by about 18 inches (45·7 cm.). The effective focal length of the combination is about 20½ feet (6·30 m.); the corrected pencil has a ratio 1 : 10, instead of the ratio 1 : 14·0 for the object-glass alone. The corrected focus is thus drawn up inside the tube of the refractor, and the collimator of the spectrograph is pushed up partly into the tube, so that the slit coincides with the corrected focus. [For some further details with regard to the correcting lens see *Monthly Notices*, vol. lvi. p. 101.]

The spectrograph is attached to the equatorial by four thumb-screws passing through the flange E of the conical gun-metal casting. The actual process of mounting the spectrograph is very easily carried out when the refractor is pointed to the zenith.

The Collimator, and Slit, and Guiding Comb.

The collimator has a focal length of 20½ inches [520 mm.] and an aperture of 2⅛ inches [54 mm.]. The effective aperture is 52 mm. with a ratio of 1 : 10.

The slit is made after the device of Sir W. Huggins; the jaws are made of spéculum metal with the front surface polished. Special care was taken to work the sharp edges in the proper manner.

A guiding comb is placed close to the slit and in front of it: it consists of a piece of thin copper foil blackened and shaped so as to have three bars or teeth which project across the slit and prevent light from entering. It is provided with mechanism (a snail cam G, &c.) for moving it through a small range along the slit. For stellar work, only two of the teeth are used. The teeth are each twice as wide as the interval between them (interval = $0^{mm}\cdot25$; teeth $0^{mm}\cdot5$). With this arrangement and three different settings of the comb nine spectra can be set side by side on the photographic plate; of these the five central ones show no overlap or superposition of spectra, whilst the two outer ones above and below may under certain circumstances of large images on the slit require special treatment to avoid overlap. The slit and guiding comb can be viewed, as from in front, by a system of mirrors and lenses which enable the observer to see either the star or the comparison spark on the slit throughout the exposures.

I have for years past adopted the method of adjusting the slit-width by the diffractional method which I have described in another paper (page 609). The following table gives the relation between the diffractional indicator and the actual slit-width in millimetres:

Diffractional indicator m =	1	2	3	4	5
Slit-width in millimetres	0·009	0·017	0·026	0·035	0·043

for $s = \frac{2m\lambda f}{a}$ and the slit has the values given for the integral values of m. The value of m when the slit subtends an angle $2''$ in the focal plane of the equatorial is $m_\gamma = 7\cdot2$. I have usually, when economy of light is of importance, adjusted the slit-width so that $m = 3$; and though I have occasionally used $m = 4$, I have never used a smaller value than $m = 1$.

It will be seen in the paper referred to, that $P = \frac{1}{2m+1} R$ and actual experiment shows that the spectra are at least as good as is indicated by this relation. Thus, the resolving power of the instrument being by calculation 76,700, the purity attained is at least as follows for the slit-widths named:

Diffractional indicator m	1	2	3	4
Purity of spectrum	25,500	15,300	10,900	8,500

The fact that these large slit-widths are practically available in stellar work proves how important a part is played by the tremor-disc. I have developed this aspect of the case fully in the paper I have just referred to (page 618), and will not enter into it further here except to point out that the installation there referred to as having an aperture of 63 cm. is to all intents and purposes the same as that of the Cambridge

Observatory, and here described, though I have reason to believe that the prism system is in reality considerably less absorbent than is there assumed. (It is, in fact, of dense flint, whereas it is there assumed to be extra dense.)

The Prism-box and Prisms.

The foundation on which the four prisms are fixed is a strong iron casting in no part less than 8 mm. thick. It is of very rigid construction, as may be inferred from Plate 19, where it is indicated in ground plan and also in section. The prisms are pressed each by a springy clip against three bosses rising from the flat cast-iron table on which they are adjusted. It was found by experiment that the coefficient of friction between the surfaces of the bosses and the ground surface of the glass prisms was nearly doubled when the bosses were lightly dusted over with fine emery. By the adoption of this plan a much lighter pressure of the clips is needed to keep the prisms fixed in position.

The prisms were worked by Hilger, all four prisms being cut out of one block of glass obtained from Jena. The angles are respectively : (1) 56° 12′, (2) 56° 4′, (3) 56° 10′, (4) 56° 11′. The prisms produce a total deviation of 180° for H_γ, when they are set to produce minimum deviation of H_γ. The refractive indices and specific gravity were determined for the prisms Nos. 1 and 2, and were found to be as follows (trustworthy to two units in the last place) :

μ_D	$\mu_{H\delta}$	$\mu_{H\gamma}$	Sp. Gr
1·6180	1·6297	1·6402	3·598

In Messrs. Schott's list of Jena glasses I find the following dense flint glasses :

O. 167	1·6169	1·6290	1·6393	3·60
O. 103	1·6202	1·6324	1·6428	3·63
O. 93	1·6245	1·6369	1·6475	3·68

and in the absence of definite information I assume that the prisms are made of Jena glass O. 167. It is a transparent glass, very slightly yellow—a point of considerable importance when four prisms, each having a base $7\frac{1}{2}$ cm. long (total base = 30·8 cm. or 12 inches), are to be used in a stellar spectrograph.

The relation connecting refractive index and wave-length in the form $\mu = a + b\lambda^{-2} + c\lambda^{-4}$ gives, for the values found here experimentally,

$$\mu = 1{\cdot}5983 + 0{\cdot}56 \times 10^{-10}\lambda^{-2} + 4{\cdot}33 \times 10^{-20}\lambda^{-4}$$

λ being expressed in centimetres

and $\frac{d\mu}{d\lambda} = 2490$

[If the values given in the Jena list are taken, $\frac{d\mu}{d\lambda} = 2390{\cdot}0$.]

In Lord Rayleigh's notation the resolving power is

$$R = (t_1 - t_2)\frac{d\mu}{d\lambda}$$

$t_1 - t_2$ being the difference in the thickness of glass traversed on the two sides of the beam, or the effective total length of base of the prism-system. The base in the present case is

$$t_1 - t_2 = 4(2 \times \text{aperture of beam} \times \sec i \sin \tfrac{a}{2})$$

a being the angle of the prisms and i the incidence of the beam upon the prism-surface. For the present purposes we may take the prism-angle as 56° 9′ and incidence as 50° 31′ for H_γ for each prism, and we have

$$\begin{aligned} t_1 - t_2 &= 4(2 \times 5{\cdot}2 \sec (50° 31') \sin (28° 4'{\cdot}5)) \\ &= 4 \times 7{\cdot}7 = 30{\cdot}8 \text{ cm.} \end{aligned}$$

Thus we have for the resolving power

$$R = 30{\cdot}8 \times 2490 = 76{,}700$$

It is of interest to compare the resolving powers of some of the larger instruments in use at other observatories. The following table has been made out from details published (1) by Professor Campbell (*Astrophysical Journal*, viii. 134), (2) by Dr. Vogel and by Dr. Hartmann (*ibid.* xi. 395, 408), (3) by Professor Frost (*ibid.* xv. 17).

	Effective Aperture of Coll. mm.	No. of Prisms.	$\mu_{H\gamma}$.	$\frac{d\mu}{d\lambda}$.	$t_1 - t_2$. Cm.	Resolving Power at $H\gamma$.
Lick Observatory	37·4	3	1·7263	3340	22·2	74,100
Potsdam „	32	3	1·6744	2780	21·3	59,200
Yerkes „	51	3	1·6766	2830	34·5	97,600
Cambridge „	52	4	1·6402	2490	30·8	76,700

The choice of glass for the prisms was decided, as is too often the case, by the existence in the hands of the optician of an actual good specimen which was accepted as "good enough." The late Mr. Adam Hilger showed me a superb block of dense flint, which he believed to have a refractive index for H_γ of $\mu = 1{\cdot}644$. It was a question whether to cut it into three prisms

or four. I decided upon four, for reasons which are summarised in the following tables, the condition of a deviation of H_γ through 180° being accepted as essential on account of mechanical arrangements :

	Refracting Angle of each Prism.	Relative Dispersion.	Length of Base for 2-inch Beam.	Total Base.
	° ′		In.	In.
4 prisms	56 0	2·95	2·96	12·0
3 prisms	64 32	3·33	4·44	13·5

Loss by reflexion at the surfaces of the prisms :—

	Losses.		Transmission.								
	$\left(\frac{\sin(i-r)}{\sin(i+r)}\right)^2$	$\left(\frac{\tan(i-r)}{\tan(i+r)}\right)^2$	1 surface.		2 surfaces.		6 surfaces.		8 surfaces.		Total.
			sin.	tan.	sin.	tan.	sin.	tan.	sin.	tan.	%
4 prisms	·153	·007	85	99	72	98			27	92	60
3 prisms	·237	·015	76	98	58	97	19	91			55

The tables showed a very even balance, and the fact that in the three-prism arrangement the axes of collimator and camera would be more widely separated than in the other was of no importance. The casting vote was given for four prisms, on the ground that there should be want of homogeneity between the top and the bottom of the slab ; then, since the prisms were all to be cut in one way from the slab, it would be safer to have an even number of prisms than an odd, so that two might be turned upside down relatively to the others. In the event, no such want of homogeneity was found to exist. The surfaces of the prisms are also excellent.

The four prisms described above are used when the spectrum in the neighbourhood of H_γ is photographed. I have used instead of the fourth prism another prism of larger deviating power, so that when the train is readjusted to produce minimum deviation of the D lines the instrument is available for photographing the green part of the spectrum between D and H_β.

The Cameras.

Three cameras are provided :

(1) a telephotographic combination specially constructed by Dallmeyer, arranged so that the effective focal length is 40 inches (1016 mm.) and the effective ratio is 1 : 20.

(2) a medium camera of focal length 20 inches (508 mm.) and of the effective ratio 1 : 10.

(3) a short camera of focal length 14 inches (356 mm.), and of the effective ratio 1 : 7.

The longest camera was used in the researches on the spectrum of *Capella* (*Monthly Notices*, vol. lx. p. 418) and of *α Persei* (*Monthly Notices*, vol. lxi. p. 12, and vol. lxii. p. 124). The medium camera has been used in determining the velocity of selected stars (*Monthly Notices*, vol. lxii. p. 296). The shortest

camera has been used for faint stars ; some of the results obtained with it are given at the end of this note.

Photographed spectra obtained with the four-prism spectrograph are numbered in a series F with a suffix to denote the camera. Thus Fl indicates long camera, Fm medium camera, Fs short camera.

The scale of the photographed spectra and the values of a single turn of the micrometer screw expressed in terms of velocity are as follows :

Wave-length.	Tenth-metres per mm.	Velocity km/sec per turn of Micrometer.	
4250	6·2	44	with the long camera (series Fl)
4300	6·7	47	
4350	7·2	49	
4200	12·2	87	with the medium camera (series Fm)
4250	13·1	92	
4300	13·9	97	
4350	14·7	101	
4400	15·6	106	
4200	19·5	139	with the short camera (series Fs)
4250	20·9	147	
4300	22·1	154	
4350	23·7	163	
4400	25·2	172	

Adjustment of the Instrument.

The only adjustment that need be referred to is that of focussing the collimator and camera. I have found the method I described (*Monthly Notices*, vol. lvii. p. 572) some years ago excellent, and have not been able to improve upon it. It is in reality only a slight modification of Schuster's method, made to meet the needs of the photographic work ; and the chief merit of the method is that it lends itself equally well to optically perfect instruments and defective instruments.

Unfortunately there were defects in the original description of the method, and these appear to have led to misunderstandings ; I should therefore like to remove them by describing it again. (i.) The collimator and camera having been approximately adjusted by visual methods, photographs are taken, three or more on a plate, with slightly different camera readings, the whole aperture being used and the collimator being left unchanged at the reading, let us suppose, of 10·0. Inspection of the photographs shows which setting of the camera gives the best definition ; let us say 16·7. (ii.) The camera is then set at 16·7, and three exposures are made on another photograph, but in this case half apertures are used by inserting diaphragms, alter-

nately to transmit (*a*) the part of the beam that passes through the thick part of the prisms and (*b*) the part that passes through the thin part of the prisms ; the photograph thus taken shows what we may call a "thick" spectrum between two "thin" spectra. It is of course necessary to use exposures about four times as long as when the whole aperture is used. If at the part of the spectrum which we desire to use for measurement there is no shift of the lines in the "thick" relatively to the "thin," then the setting of the collimator is correct. (iii.) If, however, there is a shift, then the focus of the collimator is altered, so that its reading is, let us say, 11·0 ; and with full beam the camera is refocussed by another set of photographs, and is adjusted to that reading which gives the best definition, let us say 15·4. (iv.) Then a "thick and thin" photograph is taken. Thus by a succession of trials the correct readings of collimator and camera are found.

It will be seen on comparing the photographs that whenever shift is found in the "thick and thin" exposures the lines are sharper in both the "thick" and the "thin" than they are with whole aperture ; and herein lies the value of the method, which allows us to find that adjustment of the instrument, for which all parts of the transmitted beam, whether they have passed through the thick and more absorbent part of the prism system or through the thin, come to a focus at the same spot on the photograph.

It will happen, as I have actually found it, that if for one of the prisms is substituted another prism which is either more or less defective than the one eliminated, the definition seems to have deteriorated ; and photographs may exhibit a shift of "thick" relative to "thin," which was not present previously for those readings of collimator and camera ; but unless the defect is serious, it will be possible to find a new reading of the collimator which, when the camera has been focussed with whole aperture to give the best definition, will not exhibit a shift of "thick" relative to "thin." Thus defects in one part in the system may be mitigated by what may be described as intentional maladjustment in another part. As I have indicated above, a great part of the value of the method lies in the fact that it is essentially empirical.

The shifts in the lines of the "thick" and the "thin" spectra for various collimator readings afford good evidence of the perfection or imperfection of the chromatic corrections of the collimator. And, as regards the transparency of the prism system, it may be mentioned that the exposures needed to give equal intensities at H_γ in "thick" and "thin" photographs are in the ratio of 6 : 5.

Mode of the Measurement and Reduction of the Photographs.

The photographs are measured on a Zeiss comparator, to which reference is made below, and for which a table of corrections for graduation errors is given. The plates are measured in two positions, (i.) with the spectrum so placed that the red is on the right hand (R to *r*) ; (ii.) with the plate reversed and the

spectrum "red to left" (R to *l*). In the position "red to right" the scale readings increase with increasing wave-lengths. Scale corrections are then applied. For a few plates the measures, R to *r*, and R to *l*, were separately reduced to wave-lengths by reference to three chosen lines in the iron comparison spectrum, and the results for each line were combined. For the reduction, the well-known Hartmann-Cornu relation was used. No attempt has been made to smooth the values of the wave-lengths thus deduced, by taking more lines in the comparison spectrum than the three chosen lines; and in no case has a wave-length been got by extrapolation. The reduction is done with the help of a Brunsviga calculating machine.

When three or more plates of one and the same star have been thus measured—the same lines having as far as possible been picked out in each case by identification with the help of a ten-fold enlargement on bromide paper—and when the wave-lengths have been deduced, then, assuming constant velocity in the star, a correction for the Earth's orbital motion is applied to each wave-length in the star spectrum, and thus these wave-lengths are left affected only with the velocity of the star.

The different values for each line are combined to form a mean, and this mean is used, in comparison with Rowland's solar wave-lengths, to identify, if possible, the origin of the measured line or groups of lines.

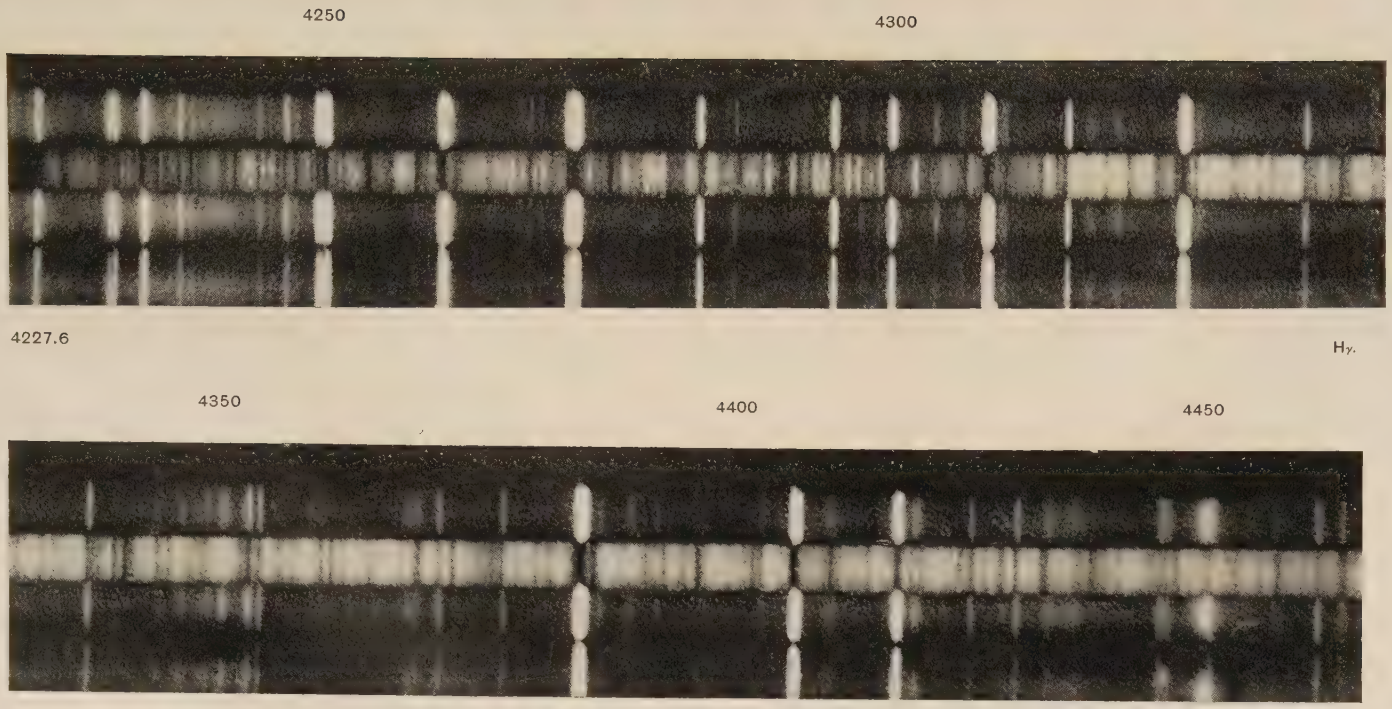
Table I. illustrates the procedure by reference to photographs of *α Boötis*. Here the numbers in any horizon in the first seven columns are the decimal figures in the wave-length for one and the same line as measured in the plates named at the tops of the columns. The mean for each line is given in the eighth column together with the wave-length digits in front of the decimal point, *e.g.* 4203·670. In the ninth column is given the solar line assumed to be of the same origin. In the last column is the difference in the sense ✱ − R.

A set of identifications carried out in this way for a few typical stars serves for many stellar spectra of corresponding type, and is not carried out for every star.

TABLE I.

Bk. LXIII. α Boötis 1902. *Measured by N.*

F 334.	F 334 bis.	F 336.	F 337.	F 338.	F 339.	F 341.	Mean.	R.'s ⊙.	✱−R.
65	64	68	69	67			4203·670	730	−060
78	76	80	82	78		78	06·787	862	−075
48	46	46	48	50		50	10·480	{494 / 561}	−047
48	46	45	46	47		47	19·465	{516 / 580}	−083
34	34	35	29	34		32	22·330	382	−052



SPECTRUM OF ARCTURUS, WITH IRON COMPARISON SPECTRUM

(Enlargement approximately Twenty-one-fold

F 334.	F 334 bis.	F 336.	F 337.	F 338.	F 339.	F 341.	Mean.	R.'s ⊙.	✱ −R.
38	41	38	31	41	41	41	45·387	{422 / 520}	−084
89	90	93	88	92	92	87	46·901	996	−095
19	12	25	21	21	23	17	50·197	287	−090
89	90	85	81	88	80	82	50·850	945	−095
40	42	41	41	41		38	54·405	505	−100
92	93	90	82	90	84	90	74·887	958	−071
24	25	19	17	18	22	14	94·200	{204 / 301}	−052
90	90	84	78	89	96	83	21·871	961	−090

Having thus picked out the solar lines, $\lambda_{\odot}$, which are to be adopted as of the same origin as those measured in the star spectra λ_{*}, the velocity for each line measured on each photograph is deduced by taking out the shift $\delta\lambda = \lambda_{*} - \lambda_{\odot}$ and multiplying it by $\frac{V}{\lambda_{\odot}}$.

Thus Tables II. and III. show the velocities deduced from each of the stellar lines, the wave-lengths of the lines being there given in the eighth column of Table I. The plate F 334 was measured twice in each position (R to *r*) and (R to *l*) both by myself (N.) and by my assistant, Mr. H. J. Bellamy (B.). These serve to show the consistency that can be obtained by one observer in different measures of the same plate, and also by two different observers. Some details have already been given in a previous note (*Monthly Notices*, vol. lxiii. p. 299) in regard to the accuracy attained in the measurement of photographs taken with this instrument.

In more recent measurements and reductions of the photographs the following procedure is adopted.

Each complete photograph has four spectra—namely, the star spectrum and three iron comparison spectra—side by side on the plate arranged thus, and photographed in the order indicated by the numbers :

3. Comparison spectrum taken after the star.
2. Star spectrum.
1. Comparison spectrum taken before the star.
4. Comparison spectrum taken last of all.

[An enlargement of one of the photographs is given in Plate 20, which has been made by photographically reducing a forty-three-fold enlargement of the original negative. The scale of the spectrum as it appears on Plate 20 is at H_{γ} nearly half that of Rowland's map of the solar spectrum.]

TABLE II.

Bk. LXIII. α Boötis 1902. *Measured by N.*

(The wave-lengths of the lines from which the following velocities were deduced are those given in the eighth column of Table I.)

F 334.		F 334 bis.		F 336.		F 337.	
7·1	5·7	7·9	2·9	17·9	16·4	20·0	16·4
5·7	7·1	5·7	3·6	17·9	15·0	20·0	15·7
10·0	7·9	7·1	7·9	18·4	14·3	17·1	17·8
9·2	6·4	7·9	4·3	17·1	10·7	17·1	12·1
8·6	8·6	10·7	7·9	20·5	16·4	15·7	12·8
8·6	3·5	9·9	5·7	14·8	13·4	13·4	4·9
8·5	1·4	9·2	2·1	18·4	14·1	14·1	10·6
4·9	5·6	8·5	7·1	19·1	16·3	15·5	14·1
9·9	7·1	10·6	7·1	16·2	12·0	13·4	8·5
5·6	3·5	7·8	3·5	16·2	12·0	16·9	11·3
10·6	8·4	10·6	9·1	19·0	14·0	13·4	8·4
14·0	9·1	14·7	9·1	18·9	13·3	15·4	13·3
10·4	4·9	10·4	4·9	15·3	9·0	10·4	4·2
Mean R to r + 9·42		+ 9·31		+ 17·81		+ 15·62	
Mean R to l	+ 6·60		+ 5·78		+ 13·65		+ 11·81
Mean for plate ...	+ 8·01		+ 7·55		+ 15·73		+ 13·71
Earth's orbital velocity	− 12·14		− 12·14		− 20·60		− 20·62
Earth's diurnal ...	− 0·14		− 0·14		− 0·21		− 0·21
Curvature correction	− 0·36		− 0·36		− 0·36		− 0·36
	− 4·53		− 4·99		− 5·44		− 7·53
		− 4·81					

The photograph having been carefully adjusted on the measuring machine so as to be parallel to the slide as judged by passing the spectrum several times through the field of view by movement of the slide, the wires of the micrometer are adjusted to parallelism with a line in the spectrum by turning the micrometer head so that the wire bisects a given line in the uppermost and lowermost spectra, 4 and 3. After this adjustment, the spectrum No. 4 is not further used : it has served its function, first, by giving a visible record, grasped at sight, of any shift between the lines in the initial and final exposures ; and secondly, by affording what is equivalent to a long line (*parts* of which appear in 4 and 3) photographed in the same state of the instrument, so

Table III.

Bk. LXII. α Boötis. Measured by B.

F 334.		F 334 bis.		F 336.		F 337.	
7·9	4·3	8·6	4·3	14·3	10·7	20·0	16·4
8·6	1·4	7·9	2·9	16·4	14·3	...	...
7·1	5·0	7·1	7·1	15·0	15·7	15·7	15·7
6·4	5·0	8·5	7·1	14·9	14·2	14·9	10·7
4·3	5·7	7·8	5·7	17·8	17·1	14·2	14·2
7·1	4·9	4·9	3·5	15·6	14·1	17·0	10·6
7·1	3·5	5·6	1·4	13·4	15·5	15·5	15·5
6·4	7·1	9·9	6·4	16·2	12·7	16·9	11·3
4·9	4·9	6·4	2·8	11·3	11·3	16·2	12·0
3·5	3·5	1·4	3·5	10·6	12·0	16·2	11·3
9·1	8·4	7·7	7·0	13·3	14·7	...	...
11·2	10·5	8·4	9·8	13·3	15·4	...	...
9·0	8·3	...	...	...	...	...	...
Mean R to r +	7·07	+ 6·97		+ 14·55		+ 16·29	
Mean R to l	+ 5·53		+ 5·05		+ 13·83		+ 13·04
Mean for plate ...	+ 6·30		+ 6·01		+ 14·18		+ 14·16
Earth's orbital velocity	− 12·14		− 12·14		− 20·60		− 20·62
Earth's diurnal ...	− 0·14		− 0·14		− 0·21		− 0·26
Curvature correction	− 0·36		− 0·36		− 0·36		− 0·36
	− 6·34		− 6·63		− 6·99		− 6·58
		− 6·48					

that the micrometer wire can be set parallel to the long line; this latter adjustment is not satisfactorily attainable with the short lines in either 1 or 3, for they are not more than $\frac{1}{4}$ mm. long.

Then the measurements are taken of the chosen lines in the iron and star spectra. The readings are taken thus: four for each standard line in 1 and 3, and two on each chosen line in the star spectrum; half of these are taken as the spectrum is pushed from right to left ("out"), and the other half as the spectrum is moved back again ("home"), so that the mean of the complete readings for each line may be considered to have been made in the mean state of the measuring instrument and of the observer. This procedure has the advantage that the two settings on each line are more completely independent of each other.

Each photograph having been measured in the two positions R to r and R to l, the readings for each line are combined in the usual manner. The Hartmann-Cornu relation is used to find λ_o, R_o, and c from three chosen standard lines in the iron comparison spectrum, and the wave-lengths of the lines in the star spectrum are deduced and are dealt with as in the example of *α Boötis* given above.

Zeiss Comparator.—The measuring machine with which measurements of the spectra have been made since September 1900 is of the smaller form of Zeiss Comparator and is marked No. 991. I received the instrument at the end of March 1900; it was accompanied by a Prüfungsschein issued by the Reichsanstalt. The silver scale is stamped with the reference figure P.T.R. II. 160.

At my request the instrument was made with the eyepieces inclined at an angle of 45° to the vertical; the observer is thus in an unconstrained position in measuring: this is not the case with instruments in which the emergent pencil of light is vertical. Furthermore, in order to set the wires of the micrometer parallel to the lines of the spectra, and also to measure the inclinations of such lines in planetary spectra, a special scale and slow motion were attached to the upper end of the microscope through which the photographs are viewed.

The instrument has a scale of length 100 mm. divided throughout its length into fifths of millimetres on silver let into brass. This scale was thoroughly investigated in the summer of 1900, and the error of each mark on the scale was determined.

The accuracy in setting on marks on the silver scale may be gauged by the fact that in ten settings the mean residual was one estimated tenth of a division on the micrometer head, corresponding to $0^{mm}{\cdot}0001$; the actual residuals were 2, 0, 0, 1, 1, 2, 0, 3, 1, 0.

The value of a micrometer revolution in different parts of the field of view was tested and found constant within the range used in the measurements.

Errors in the micrometer screw were looked for by measuring a small interval, equal to twenty-five divisions on the micrometer head, in twenty-four positions round the micrometer head, with the following results:

Magnitude of residuals in estimated tenths ...	0	1	2	3	4	5	6	7	8	9	10
No. of residuals ...	3	5	2	5	4	3	0	0	2	0	0

The screw has accordingly been assumed free from error.

With a view of testing the accuracy with which the scale is subdivided into equal parts, the following procedure was adopted.

(1) The error of the middle mark 50·0 of the scale was carefully determined by measurements of a glass scale (made by

Zeiss) in reversed positions. (2) The intervals between the "ten" marks 0·0, 10·0, 20·0 . . . 100·0 were then compared with a specially ruled interval on glass scale; and the error of each of these marks was so determined. (3) The intervals between the whole millimetre marks 0·0, 1·0, 2·0 . . . 100·0 were measured on the micrometer screw, and the error of each of these marks was so determined. (4) And lastly, the intervals between the successive marks 0·0, 0·2, 0·3 . . . 100·0 were measured on the micrometer screw.

The following table thus represents the corrections to be applied to the micrometer readings for each one of the 500 marks on the silver scale used in settings. The corrections for the "ten" marks 0·0, 10·0, 20·0 . . . and also for the whole millimetre marks are based on my own measurements. The corrections for the $\frac{1}{5}$th-millimetre marks are based on means of separate observations made by myself and my assistant, Mr. A. W. Goatcher, who is now at the Royal Observatory, Cape of Good Hope.

Table of Corrections.

	0	10	20	30	40	50	60	70	80	90
0·0	0	− 8	+ 9	+21	+22	+18	+18	+12	+ 1	− 4
2	+ 8	+ 9	+17	+25	+37	+17	+31	+19	− 4	+ 1
4	0	+ 6	− 4	+11	+45	+23	+29	+10	− 6	− 2
6	+ 1	+ 7	+ 4	+22	+26	+17	+27	+24	− 7	+12
8	− 9	+ 9	− 9	+11	+28	+19	+21	+ 8	−10	+11
1·0	0	+ 5	+ 7	+12	+23	+20	+12	+13	− 4	0
2	+ 9	0	− 2	+25	+30	+23	+25	+11	+ 3	− 1
4	0	+ 1	+ 8	+26	+42	+17	+ 8	+11	0	+ 8
6	+ 1	− 5	+ 9	+30	+29	+25	+14	+16	−11	− 3
8	+ 1	+ 2	+ 6	+16	+26	+16	+ 7	+14	+ 3	− 8
2·0	+ 7	− 3	+17	+19	+27	+23	+ 7	+15	0	+ 5
2	+ 3	+ 8	+21	+17	+28	+32	+10	+14	+ 2	+ 8
4	+ 3	+ 4	+12	+ 4	+15	+26	+ 6	+ 5	− 4	− 9
6	− 1	+ 4	+11	+13	+25	+21	+ 6	+ 7	− 7	−10
8	+ 7	+ 4	+ 7	+22	+15	+22	− 1	+20	+ 2	− 2
3·0	0	0	+ 6	+19	+19	+21	+ 6	+17	− 3	− 2
2	+11	+ 8	+ 6	+25	+27	+29	+26	+ 2	+13	−17
4	+10	+ 7	− 9	+18	+28	+36	+17	+10	+15	− 7
6	+ 7	+ 2	0	+21	+24	+38	+36	+ 8	+10	− 3
8	− 2	+ 2	− 5	+21	+25	+23	+16	− 5	+ 9	−10

	0	10	20	30	40	50	60	70	80	90
4·0	+ 2	+ 4	− 3	+11	+13	+32	+ 4	+ 3	+12	− 4
2	+ 3	+10	+16	+16	+11	+39	+ 5	+17	+16	− 4
4	− 2	− 6	− 1	+10	+ 2	+29	− 2	+22	+17	− 6
6	− 6	+11	+ 5	+11	+ 4	+24	+ 3	0	+20	−11
8	− 6	− 6	+ 8	+ 9	+ 2	+38	+18	+ 8	+11	− 9
5·0	− 7	+ 2	+ 8	+10	+ 9	+31	+17	+ 6	0	−15
2	−12	+21	+17	+ 2	+31	+31	+23	+ 5	+ 7	+ 2
4	−11	+ 6	+11	+14	+18	+35	+18	+19	+ 3	− 4
6	+ 2	+11	+ 5	+12	+31	+36	+27	+12	+ 6	+ 6
8	− 1	+11	+ 9	+11	+32	+33	0	− 9	+ 8	− 5
6·0	−12	+ 5	+ 2	+14	+30	+19	+17	+ 4	+ 8	− 5
2	− 3	− 5	+ 5	+26	+30	+23	+19	+ 6	+ 3	+ 1
4	− 8	− 3	−11	+18	+17	+21	+ 9	+ 9	+ 5	+ 3
6	− 1	+ 1	+ 7	+15	+12	+24	+ 6	+ 3	+ 6	+ 1
8	+ 9	+ 7	− 8	+21	+21	+21	+ 4	− 4	+ 2	+12
7·0	−12	− 7	+ 5	+ 4	+25	+23	+ 1	+ 5	− 8	+ 8
2	−16	−10	+10	+ 4	+ 7	+21	− 4	0	+ 2	+ 6
4	− 4	−14	− 3	+ 9	+ 6	+19	+ 1	+13	− 9	− 3
6	−17	− 8	− 1	+18	+16	+13	+ 2	+ 9	− 8	− 8
8	− 9	− 1	+15	+ 7	+20	+40	− 7	0	− 2	−11
8·0	−13	− 7	+ 7	+16	+13	+17	− 8	− 2	− 6	+ 7
2	− 9	− 2	+17	+ 6	+25	+29	− 7	− 6	+ 5	+11
4	− 7	−20	+27	+17	+11	+28	+ 7	− 8	− 1	− 4
6	− 6	− 8	+15	+ 4	+15	+19	+18	− 8	0	+14
8	−11	− 4	+19	+ 7	+25	+18	− 2	− 6	+ 1	+12
9·0	−15	+ 4	+25	+ 5	+14	+14	− 2	− 7	− 6	− 2
2	−15	+11	+29	+15	+18	+18	+10	− 4	0	− 8
4	−30	+10	+26	+ 4	+16	+30	+25	− 7	− 3	− 5
6	−13	+ 4	+27	+ 8	+21	+15	+21	+12	−12	− 8
8	−21	0	+19	+26	+29	+19	+32	+ 2	0	− 2

The following note contains some of the results obtained in the year 1903 in continuation of those already communicated to the Society (*Monthly Notices*, vol. lxiii. p. 299).

Velocity in the Line of Sight. Selected Stars. Cambridge Observatory, II. 1903. By H. F. Newall.

The present note is a second contribution to the plan of co-operation between certain observatories to determine the velocity in the line of sight of selected stars. Since the first contribution (*Monthly Notices*, vol. lxiii. p. 296) no alterations have been made in the instrument, but a fuller description of it than has yet been given is published in the present number of the *Monthly Notices*, p. 636. Fuller details of the measurements will, it is hoped, shortly appear in the *Astrophysical Journal*; only the results of the measurements are given here, together with a summary of the mean velocities of the nine stars dealt with.

The year 1903 was unfavourable for observations; the brighter stars were often accessible only by longer exposures than usual, but the photographs of the spectra of fainter stars could hardly be obtained with the linear dispersion given by the four prisms and the medium camera (focal length 520 mm.). As it was regarded as particularly desirable to get spectra of the fainter stars on the list such as β *Ophiuchi* (Draper Catalogue magnitude at Hγ 4·19), γ *Aquilæ* (D.C.M. 4·66), and γ *Piscium* (D.C.M. 5·03), the shorter camera (Dallmeyer Doublet, focal length 356 mm.), was put on in place of the medium. My assistant, Mr. Bellamy, made the most praiseworthy efforts to secure measurable photographs, and succeeded in getting six photographs of β *Ophiuchi* and γ *Aquilæ*; but the sky was never clear enough to make it worth while to attempt to secure photographs of γ *Piscium*.

The velocities deduced for β *Ophiuchi* and γ *Aquilæ* must be regarded as of inferior weight; for the photographs are weak even compared with those got since (in 1904) with the medium camera.

The measurements have been made under my directions by Mr. Bellamy, illness having prevented me from carrying out my intention of making the measurements in duplicate. I am much indebted to Mr. Bellamy for his continued efforts under circumstances that must have been peculiarly discouraging to him.

In the record of the photographs given below

Fm denotes those taken with the medium camera (520 mm.)

Fs „ „ „ shorter „ (356 mm.)

The velocities deduced in the Fs series seem to be more or less consistently about 2–3 km/sec lower than those deduced in the Fm series. I have not been able to assign a cause for this, and have accordingly given the results as deduced from photographs obtained with all due care. The difference appears also in the case of α *Boötis*, though the results are not given below.

It should be stated that the instrument is not provided with electrical temperature control, but is only encased in a thick quilted feather cover. As instances of the success of this cover, the following records are of interest, though the electrical control would give results of a different order :

No. of Photograph.	Month.	Exposure.	Temperature on case of Prism-box. Beginning of Exposure.	End of Exposure.
		m	°	°
Fm 394	Feb.	70	11·7 C.	11·3
Fm 413	Mar.	50	4·1	3·9
Fm 415	Mar.	120	5·6	4·5
Fm 422	Apr.	22	7·0	7·1
Fm 425	Apr.	25	7·2	7·1
Fm 440	May	70	11·7	11·3
Fs 482	July	67	12·0	11·3
Fs 487	Aug.	80	16·9	16·0
Fs 488	Aug.	75	15·5	15·0
Fs 505	Oct.	80	9·8	9·5
Fs 525	Nov.	28	6·8	6·5
Fs 534	Dec.	50	0·3	0·3
Fs 540	...	37	3·2	3·0

A special temperature control has not been fitted to the instrument, not because I think it unnecessary, but because I had the intention to remodel the whole instrument ; and I desired to give a trial of a plane-grating spectrograph [which is being constructed and is nearly ready for use] before doing this.

Star and No. of Plate.	Date and G.M.T. Mid-exposure.		Expo-sure.	Hour-angle.	Slit-width.	Range of Spectrum.	Comp. Spect.	No. of Lines.	Velocity relative to Earth.	Velocity reduced to Sun.
α Arietis, 1903.		h m	m		mm.				km/sec.	km/sec.
Fs 494	Sept. 26	9 56	43	4 11 E	0·025 ($m=3$)	4202 4405	Fe Spk.	10	−30·27	−14·23
Fs 504	Oct. 13	11 17	45	1 44 E	,,	,,	,,	15	−21·47	−13·30
Fs 506	16	12 0	50	0 52 E	,,	,,	,,	14	−22·59	−15·94
Fs 508	21	8 55	40	3 32 E	,,	,,	,,	14	−21·41	−17·05
Fs 510	23	11 48	47	0 34 E	,,	,,	,,	13	−18·79	−16·67
Fs 513	28	9 20	50	2 42 E	,,	,,	,,	13	−18·93	−18·25
Fs 534	Dec. 1	10 45	50	0 57½W	,,	,,	,,	13	+ 0·39	−15·79
Fs 540	4	8 38	37	0 54 E	,,	,,	,,	13	− 2·39	−19·63
Mean	1903·423							(8 photographs)		−16·36

r and o. of ate	Date and G.M.T. Mid-exposure.			Expo-sure.	Hour angle.		Slit-width.	Range of Spectrum.		Comp. Spect.	No. of Lines.	Velocity relative to Earth.	Velocity reduced to Sun.	Mean Error.
Persei, 1903.		h	m	m			mm.					km/sec.	km/sec.	ϵ_o.
498	Oct. 5	11	51	32	2	48 E	0·025 ($m=3$)	4202	4405	Fe Spk.	14	−21·18	− 1·85	±0·84
501	12	8	39	28	5	30 E	,,	,,		,,	15	−23·37	− 4·11	±1·19
507	21	7	59	28	5	35 E	,,	,,		,,	19	−18·88	− 4·78	±1·25
525	Nov. 14	12	14	28	0	15 W	,,	,,		,,	16	− 5·81	− 1·95	±1·03
548	Dec. 14	7	18	26	2	42 E	,,	,,		,,	12	+ 2·26	− 7·33	±0·98
,,	(remeasured)											+ 2·23	− 7·36	±0·58
Mean	**1903·832**										(5 photographs)		**− 4·56**	p.e. ±0·71
Geminorum, 1903.														
n 394	Feb. 9	8	0	70	2	23 E	0·025 ($m=3$)	4202	4326	Fe Spk.	13	+16·68	+ 3·04	±0·68
n 410	Mar. 6	11	50	40	3	5 W	0·017 ($m=2$)	,,		,,	13	+23·76	+ 0·06	±0·41
n 413	7	12	10	50	3	29 W	,,	,,		,,	14	+26·09	+ 1·95	±0·40
n 414	16	8	25	40	0	20 W	,,	,,		,,	12	+28·01	+ 1·70	±0·46
n 419	21	10	12	42	2	25 W	,,	,,		,,	13	+29·89	+ 2·38	±0·61
547	Dec. 11	11	45	40	2	36 E	0·025 ($m=3$)	,,		,,	9	−14·88	+ 2·61	±1·62
Mean	**1903·306**										(6 photographs)		**+ 1·96**	p.e. ±0·29
Boötis, 1903.														
n 422	Apr. 4	11	39	22	1	44 E	0·017 ($m=2$)	4202	4326	Fe Spk.	14	−11·71	− 7·25	±0·58
n 423	,, 7	11	0	20	2	11 E	,,	,,		,,	14	−10·20	− 7·00	±0·67
n 424	,, 8	11	10	20	1	57 E	,,	,,		,,	14	− 9·07	− 6·33	±0·52
n 425	,, ,,	11	57	25	1	10 E	,,	,,		,,	14	− 9·79	− 7·10	±0·41
n 428	,, 20	10	57	23	1	23 E	,,	,,		,,	13	− 3·65	− 6·35	±1·15
n 432	,, 22	12	47	15	0	35 W	,,	,,		,,	13	− 3·10	− 6·64	±0·71
n 435	,, 24	12	49	22	0	45 W	,,	,,		,,	14	− 1·60	− 6·02	±0·81
n 436	,, 27	11	5	20	0	47 E	,,	,,		,,	14	− 2·89	− 8·43	±0·61
n 437	,, ,,	11	56	21	0	4 W	,,	,,		,,	14	− 1·70	− 7·32	±0·50
n 438	,, ,,	12	53	22	1	1 W	,,	,,		,,	14	− 0·76	− 6·47	+0·63
n 443	May 4	11	59	19	0	35 W	,,	,,		,,	12	+ 4·62	− 3·93	±0·55
n 446	,, 7	13	0	14	1	48 W	,,	,,		,,	13	+ 4·79	− 5·02	±0·62
n 448	,, 12	12	27	14	1	34 W	,,	,,		,,	10	+ 6·75	− 5·01	±0·77
n 449	,, 15	12	5	24	1	24 W	,,	,,		,,	14	+ 7·49	− 5·37	±0·41
n 452	,, 20	9	51	17	0	30 E	,,	,,		,,	14	+ 6·03	− 8·46	±0·79

Star and No. of Plate.	Date and G.M.T. Mid-exposure.	Expo-sure.	Hour-angle.	Slit-width.	Range of Spectrum.	Comp. Spect.	No. of Lines.	Velocity relative to Earth.	Velocity reduced to Sun.	Mean Error
α Boötis, 1903.	h m	m		mm.				km/sec.	km/sec.	ϵ_o.
Fm 458	May 25 12 52	35	2 48 W	0·017 ($m=2$)	4202 4326	Fe Spk.	14	+ 8·31	− 7·76	± 0·5
Fm 461	„ 26 11 13	14	1 15 W	„	„	„	13	+ 11·37	− 4·11	± 0·5
Fm 464	„ 27 9 59	18	0 5 W	„	„	„	14	+ 8·56	− 8·30	± 0·6
Fm 466	June 2 9 44	18	0 14 W	„	„	„	13	+ 10·44	− 8·22	± 0·8
										p.e.
Mean **1903·333**							(19 photographs)		**− 6·58**	± 0·21
β Ophiuchi, 1903.										
Fs 482	July 7 13 18	67	2 38 W	0·025 ($m=3$)	4202 4326	Fe Spk.	6	− 7·46	− 16·81	± 4·2
Fs 487	Aug. 4 10 25	80	1 35 W	„	„	„	7	+ 4·43	− 14·89	± 1·6
Mean **1903·551.**							(2 photographs)		**− 15·85**	
γ Aquilæ, 1903										
Fm 473	June 22 13 5	70	0 37 E	0·017 ($m=2$)	4202 4405	Fe Spk.	7	− 11·37	+ 0·87	± 3
Fs 485	July 24 11 45	70	0 44 W	0·025 ($m=3$)	„	„	11	− 1·94	− 2·71	± 1
Fs 488	Aug. 4 11 50	75	1 34 W	„	„	„	8	+ 3·14	− 2·31	± 2
Fs 492	„ 7 11 15	78	1 13 W	„	„	„	10	+ 3·35	− 3·32	± 1
Mean **1903·555**							(4 photographs)		**− 1·87**	
ε Pegasi, 1903										
Fs 502	Oct. 12 10 30	70	2 13 W	0·025 ($m=3$)	4260 4405	Fe Spk.	11	+ 23·68	+ 2·73	± 1
Fs 505	16 10 5	80	2 2 W	„	4202 4405	„	17	+ 27·07	+ 4·90	± 1
Fs 512	26 8 40	70	1 18 W	„	„	„	16	+ 26·99	+ 2·26	± 1
Mean **1903·795**							(3 photographs)		**+ 3·30**	
ε Leonis, 1903										
Fm 415	Mar. 16 10 50	120	0 44 W	0·017 ($m=2$)	4202 4326	Fe Spk.	15	+ 21·39	+ 4·47	± 0
Fm 434	Apr. 24 12 15	90	4 42 W	„	„	„	11	+ 30·12	+ 1·95	±
Fm 440	May 2 9 20	70	2 19 W	„	„	„	9	+ 33·93	+ 5·04	±
Fm 447	12 9 42	65	3 20 W	„	„	„	6	+ 31·30	+ 1·88	±
Mean **1903·303**							(4 photographs)		**+ 3·34**	
ε Virginis, 1903										
Fm 444	May 4 13 45	80	3 27 W	0·017 ($m=2$)	4202 4326	Fe Spk.	12	+ 0·31	− 15·76	±
Fm 445	7 11 48	76	1 50 W	„	„	„	13	+ 2·70	− 13·81	±
Mean **1903·342**							(2 photographs)		**− 14·78**	

Summary.

Epoch.	Star.	No. of Photographs.	Velocity in Line of Sight.
1903·423	α Arietis	8	−16·36 ± 0·49
1903·832	α Persei	5	− 4·56 ± 0·71
1903·306	β Geminorum	6	+ 1·96 ± 0·29
1903·333	α Boötis	19	− 6·58 ± 0·22
1903·551	β Ophiuchi	2	−15·85
1903·555	γ Aquilæ	4	− 1·87
1903·795	ϵ Pegasi	3	+ 3·30
1903·303	ϵ Leonis	4	+ 3·34
1903·342	ϵ Virginis	2	−14·78

Errata in Mr. Nevill's Paper.

Monthly Notices, vol. lxv. p. 267, first three lines.

For −0·51, −1·01, −1·83
read +0·51, +1·01, +1·83.